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The role of universal and non universal Sudakov logarithms in four fermion processes at TeV energies: the one-loop approximation revisited.*

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Abstract

We consider the separate effects on four fermion processes, in the TeV energy range, produced at one loop by Sudakov logarithms of universal and not universal kind, working in the 't Hooft $\xi = 1$ gauge. Summing the various vertex and box contributions allows to isolate two quite different terms.The first one is a combination of vertex and box quadratic and linear logarithms that are universal and independent of the scattering angle θ. The second one is θ-dependent, not universal, linearly logarithmic and only produced by weak boxes. We show that for several observables, measurable at future linear e^+e^- colliders (LC, CLIC), the role of the latter term is dominant and we discuss the implications of this fact for what concerns the reliability of a one-loop approximation.

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I. INTRODUCTION.

The fact that double logarithms of the c.m. energy $\sqrt{q^2}$ of "Sudakov-type" [1] will affect the asymptotic behaviour of the weak component of four-fermion processes at one-loop has been known since a decade [2]. Only recently, though, it has been realized that this effect could be already relevant for c.m. energies in the TeV range [3], which is the case of the next generation of linear e^+e^- colliders at 500 GeV (LC) [4] and 3 TeV (CLIC) [5]. In fact, a first explicit calculation of the double logarithms effects [6] showed that their size at TeV energies might well cross the relative ten percent value. In a following paper [7], the extra effects at one loop of the subleading linear logarithms on a class of observables (cross sections, asymmetries) were thoroughly computed, showing the existence of a certain cancellation between the leading and the subleading terms in a "pre-asymptotic" energy region, whose details strongly depend on the considered final state and observable [8,9]. As a general conclusion, it was stressed in Refs. [6–9] that the validity of a one loop approximation for four-fermion processes in the TeV region was not obvious, and seemed also to depend strongly on the particular chosen observable.

To get rid of low convergence rate in the perturbative expansion, several papers were written in very recent times with the specific aim of resumming the dangerous leading components of the Sudakov logarithms to all orders [10], or at least to compute them at two loops [11]. Since, as already noticed in Ref. [7], the role of the subleading logarithms is crucial already at the one loop level, the next effort was that of trying to resum to all orders both leading and subleading Sudakov logarithms. This task has been performed in at least two recent papers [12,13]. The conclusion of Ref. [13] is that the set of Sudakov logarithms can be divided into two subsets, that can be distinguished at the level of invariant scattering amplitude since the first subset provides a universal contribution that is only dependent on the c.m. energy $\sqrt{q^2}$, whilst the second, not universal one, is not only depending on $\sqrt{q^2}$ but also on the scattering angle θ. While for the first subset precise prescriptions exist that allow to resum to all orders the involved logarithms, for the θ-dependent part a clean resummation prescription does not seem to exist at the moment [13]. Should the role of this angular component become relevant for some specific observables, the problem of, at least, computing it to the next two loop level might arise for the purpose of a high precision determination of the Standard Model component of those quantities, and this calculation does not appear to be an easy task.

The aim of this paper is precisely that of investigating the role of the θ-dependent non universal components of the Sudakov logarithms, via an exhaustive analysis of the size of their effect on the class of experimental observables that will be measured at the next linear electron-positron colliders, LC and CLIC. This analysis will be performed at the one loop level, under realistic assumptions concerning the available experimental accuracy of these machines. Our philosophy will be that of assuming that any gauge invariant effect that is "tolerable", given the expected experimental accuracy, at the one loop level, can safely be computed in that approximation, thus avoiding the hard problem of a two loop analysis. We shall be working in the familiar 't Hooft $\xi = 1$ gauge, where all the Sudakov logarithms at one loop are produced by either vertices or boxes. Our analysis will only consider what we call "genuine" weak effects, due to exchanges of virtual $W's$ and $Z's$ (and, possibly, would-be Goldstone bosons for final "massive" quarks). All other diagrams will be considered as

belonging to the "QED" component, and not discussed in this paper. For instance, we shall not include in our analysis the boxes with one photon and one Z as done in Ref. [12]. At the one loop level, this will create very small numerical differences that will not change any of our conclusions, as we will see later on.

Technically speaking, the paper will be organized as follows. In Section II we shall discuss the way in which the θ-independent terms always combine in a special expression $U(q^2) = 3\ln q^2 - \ln^2 q^2$. This is precisely the combination which was shown to exponentiate in the massless quarks approximation [13]. The remaining Sudakov contributions are those of Yukawa origin ($\simeq m_t^2, m_b^2$) and the θ-dependent ones. In Section III, we discuss the different roles of the various terms in the various experimental observables, with special emphasis on the two LC and CLIC situations. We shall see that there are energies and observables where the role of the θ-independent terms can be totally neglected, so that the bulk of the Sudakov effects at one loop is given by the corresponding angular dependent part of the "genuine electroweak" boxes. In the final Section IV, we shall discuss the validity of the one loop approximation for various observables at various energies. In particular, we shall try to clarify the role of some approximations that were used in our approach and also in other, similar ones, with which we shall compare our numerical results. This should allow us to draw a number of general conclusions. A short Appendix A will be devoted to an investigation of the Sudakov effect in the special case of forward-backward asymmetries, that seem to have a peculiar role in this case. In Appendix B we give the general form of the polarized differential cross sections in our theoretical scheme, from which it is easy to get the prediction for all the considered observables.

II. UNIVERSAL AND NON UNIVERSAL SUDAKOV LOGARITHMS AT ONE LOOP

We shall only consider in this paper the "genuinely electroweak" one-loop component of the invariant scattering amplitude for the process of electron-positron annihilation into a $f\bar{f}$ fermion-antifermion pair, where the mass of the fermion f will be retained in the two cases $f = b, t$. This means that we shall only consider those one-loop diagrams that add to the Born structure virtual exchanges of weak particles(i.e. W, Z, Φ, H, where $\Phi^{\pm,0}$ are the would-be Goldstone bosons and H the physical Higgs boson). In particular, the boxes with one photon and one Z will be considered as belonging to the "QED" component, to be computed separately. This is, at one loop, a mere conventional division, that appears to us justified to the extent that such boxes must be in any case combined with initial and final photon emission interference in existing computational programs [14].

For what concerns the practical approach, we shall follow the prescription originally proposed a few years ago [15] and called "Z-peak subtracted". In this approach, the invariant amplitude is decomposed in four terms, corresponding to the four independent Lorentz structures of the process, denoted as:

$$\mathcal{M}^{(1)}_{lf}(q^2,\theta) = \mathcal{M}^{(1)}_{lf,\gamma\gamma}(q^2,\theta) + \mathcal{M}^{(1)}_{lf,ZZ}(q^2,\theta) + \mathcal{M}^{(1)}_{lf,\gamma Z}(q^2,\theta) + \mathcal{M}^{(1)}_{lf,Z\gamma}(q^2,\theta) \qquad (2.1)$$

where the (γ, Z) indices denote projections of the external Lorentz vectors and axial vectors of the process on the photon and Z structures of the initial and final particles (for

example , $\mathcal{M}^{(1)}_{lf,\gamma\gamma}(q^2,\theta)$ and $\mathcal{M}^{(1)}_{lf,ZZ}(q^2,\theta)$ have the same Lorentz structure as the photon and Z exchanges at Born level). The technical features have been exhaustively discussed in previous recent references [7,8], to which we defer the reader for more details. Here we shall only repeat that to each of the four independent Lorentz structures, that must be by construction evidently gauge-independent, there corresponds at one loop a certain "form factor", depending on q^2 (the squared c.m. energy) and θ (the c.m. scattering angle). This consists of a precise linear combination of self-energies, vertices and boxes that must be, consequently, gauge-independent, as one can easily verify e.g. by following the fundamental Degrassi-Sirlin approach [16] to which, as usually, we shall stick in this paper.

Following our previous definitions, we shall call $\tilde{\Delta}_{\alpha,lf}(q^2,\theta)$, $R_{lf}(q^2,\theta)$, $V_{\gamma Z,lf}(q^2,\theta)$, $V_{Z\gamma,lf}(q^2,\theta)$ these four form factors. To derive the corresponding contributions from self-energies, vertices and boxes is straightforward once the "projection" operations have been defined [15]. This allows to derive in an easy way the related contributions to each observable quantity of the process, starting from the definition of differential cross sections that can be found e.g. in the Appendix B of Ref. [7] and that we also repeat, for sake of completeness, in the Appendix B of this paper.

After this short and unavoidable preliminary summary, we are now ready to discuss the contributions of the various one-loop diagrams to the asymptotic Sudakov logarithms, keeping in mind the fact that our treatment is performed in the t'Hooft $\xi = 1$ gauge. Quite generally, we shall first divide the contributions into those of "universal" and those of "non universal" type. Following the conventionally adopted definitions [13], we shall call "universal" those corrections that only depend on the quantum numbers of the <u>external</u> lines, but are otherwise process-independent, and "not universal" the remaining terms. In practice, the first set will contain all those contributions that are only depending on q^2, and do not depend on θ. These will always come from vertices and, partially, from boxes. The second, θ-dependent set, will be provided by a certain component of the box diagrams, both with WW and with ZZ exchange, although the latter ones will be numerically much smaller in the SM. Note that we shall consider, at least for a first approach, production of "physical" (i.e. <u>not chiral</u>) fermions, considering observable quantities where one sums over the final spins. A restriction to a fixed final chirality is obviously straightforward using the rules for projecting $\gamma^\mu P_L$ and $\gamma^\mu P_R$ terms on the photon and Z Lorentz structure [15].

The Feynman diagrams that produce Sudakov logarithms are shown in Fig.1. Rather than writing their explicit contributions to the components (vertices, boxes) of the scattering amplitude, we shall review here those to the four independent form factors that we are using. More precisely, we find the following results:

a) Vertex with one W (Fig.1a). This produces a universal contribution, always proportional to the combination

$$U_W(q^2) = 3ln\frac{q^2}{M_W^2} - ln^2\frac{q^2}{M_W^2} \tag{2.2}$$

b) Vertex with one Z (Fig.1b). This generates a similar universal contribution, proportional to the combination

$$U_Z(q^2) = 3ln\frac{q^2}{M_Z^2} - ln^2\frac{q^2}{M_Z^2} \tag{2.3}$$

Clearly, in an asymptotic energy regime one will be fully entitled to write

$$U_W(q^2) \simeq U_Z(q^2)$$

if only the leading squared logarithm has to be retained. For a more rigorous selection that also includes the subleading linear logarithms this is, though, not correct and we shall treat separately the two effects in our analysis.

c) Vertex with two $W's$ (Fig.1c). This provides a universal Sudakov term that is of subleading (linearly logarithmic) type. Apparently this breaks the feature of producing a $U_W(q^2)$ term like that of eq.(2.2). In fact, in the $\xi = 1$ gauge, this term is reproduced at one loop by taking into account the additional contributions coming from boxes with two $W's$ (W box), as we shall now discuss.

d) W box (Fig.1d). This produces two quite different Sudakov terms. The first one is of leading (quadratic) logarithmic type, is universal and is only q^2 dependent. When added to the universal and subleading logarithmic term produced by the vertex with two $W's$, it recomposes the combination $U_W(q^2)$ of Ref. [13]. The second contribution is non universal, θ-dependent and of subleading (linear) logarithmic type. This, in a general analysis like that of Ref. [13], would not exponentiate.

e) Z boxes (Fig.1e). These only produce a non universal, θ-dependent subleading linear logarithm. Generally speaking, the size of this term, that would also not exponentiate, is much smaller than that of the corresponding one produced by the W box.

f) Would-be Goldstone bosons and Higgs vertices (Fig.1f). These add extra universal, θ-independent, subleading Sudakov logarithms of Yukawa type, that only affect final $b\bar{b}$ and $t\bar{t}$ production. In the treatment given in Ref. [13] they are grouped together with the set of θ-independent logarithms $\simeq U_W(q^2)$,$U_Z(q^2)$ and exponentiate cumulatively with them.

Our results are exposed in analytic form in the next equations, that provide the expressions of the four gauge-independent form factors for final states $\mu^+\mu^-$, $u\bar{u}$, $d\bar{d}$. The extra Yukawa terms that appear must only be retained when $u = t$, $d = b$. In our classification, we have first listed the θ-independent contributions, putting the one W and one Z vertices in the first two terms and the universal contribution produced by the combination of the W boxes with the $2W$ vertex in the third term; θ-dependent non universal W and Z boxes follow in the next terms, and the universal θ-independent Yukawa contributions appear in the last term. The obtained expressions are the following ($S \equiv$ Sudakov)

$$\tilde{\Delta}^S_{\alpha,l\mu}(q^2,\theta) = \frac{\alpha(1-v_e^2)}{32\pi s_W^2 c_W^2}U_Z(q^2) + \frac{\alpha}{2\pi}U_W(q^2) \\ -\frac{\alpha}{\pi}ln[\frac{1-cos\theta}{2}]ln\frac{q^2}{M_W^2} + \frac{\alpha(1-v_e^2)^2}{256\pi s_W^4 c_W^4}ln[\frac{1+cos\theta}{1-cos\theta}]ln\frac{q^2}{M_Z^2} \tag{2.4}$$

$$R^S_{l\mu}(q^2,\theta) = \frac{\alpha}{4\pi s_W^2}U_W(q^2) - \frac{\alpha(1+3v_e^2)}{32\pi s_W^2 c_W^2}U_Z(q^2) - \frac{\alpha c_W^2}{2\pi s_W^2}U_W(q^2) \\ +\frac{\alpha c_W^2}{\pi s_W^2}ln[\frac{1-cos\theta}{2}]ln\frac{q^2}{M_W^2} - \frac{\alpha v_e^2}{4\pi s_W^2 c_W^2}ln[\frac{1+cos\theta}{1-cos\theta}]ln\frac{q^2}{M_Z^2} \tag{2.5}$$

$$V^S_{\gamma Z,l\mu}(q^2,\theta) = V^S_{Z\gamma,l\mu}(q^2,\theta) = \frac{\alpha}{8\pi s_W c_W}U_W(q^2) - [\frac{\alpha v_e(1-v_e^2)}{128\pi s_W^3 c_W^3} + \frac{\alpha v_e}{8\pi s_W c_W}]U_Z(q^2)$$

$$-\frac{\alpha c_W}{2\pi s_W}U_W(q^2)+\frac{\alpha c_W}{\pi s_W}ln[\frac{1-cos\theta}{2}]ln\frac{q^2}{M_W^2}-\frac{\alpha v_e(1-v_e^2)}{32\pi s_W^3c_W^3}ln[\frac{1+cos\theta}{1-cos\theta}]ln\frac{q^2}{M_W^2} \quad (2.6)$$

$$\begin{aligned}\tilde{\Delta}^S_{\alpha,lu}(q^2,\theta)=&-\frac{\alpha}{12\pi}U_W(q^2)+\frac{\alpha(2-v_e^2-v_u^2)}{64\pi s_W^2c_W^2}U_Z(q^2)+\frac{\alpha}{2\pi}U_W(q^2)\\&-\frac{\alpha}{\pi}ln[\frac{1+cos\theta}{2}]ln\frac{q^2}{M_W^2}-\frac{3\alpha(1-v_e^2)(1-v_u^2)}{512\pi s_W^4c_W^4}ln[\frac{1+cos\theta}{1-cos\theta}]ln\frac{q^2}{M_Z^2}\\&-\frac{\alpha}{24\pi s_W^2}\ln\frac{q^2}{m_t^2}\left[(3-2s_W^2)\frac{m_t^2}{M_W^2}+2s_W^2\frac{m_b^2}{M_W^2}\right]\end{aligned} \quad (2.7)$$

$$\begin{aligned}R^S_{lu}(q^2,\theta)=&\frac{\alpha}{4\pi s_W^2}(1-\frac{s_W^2}{3})U_W(q^2)-\frac{\alpha(2+3v_e^2+3v_u^2)}{64\pi s_W^2c_W^2}U_Z(q^2)-\frac{\alpha c_W^2}{2\pi s_W^2}U_W(q^2)\\&+\frac{\alpha c_W^2}{\pi s_W^2}ln[\frac{1+cos\theta}{2}]ln\frac{q^2}{M_W^2}+\frac{\alpha v_ev_u}{4\pi s_W^2c_W^2}ln[\frac{1+cos\theta}{1-cos\theta}]ln\frac{q^2}{M_Z^2}\\&+\frac{\alpha}{16\pi s_W^2}\ln\frac{q^2}{m_t^2}\left[\left(1+\frac{4s_W^2}{3}\right)\frac{m_t^2}{M_W^2}+\left(1-\frac{4s_W^2}{3}\right)\frac{m_b^2}{M_W^2}\right]\end{aligned} \quad (2.8)$$

$$\begin{aligned}V^S_{\gamma Z,lu}(q^2,\theta)=&\frac{\alpha(3+2c_W^2)}{24\pi s_Wc_W}U_W(q^2)-[\frac{\alpha v_e(1-v_e^2)}{128\pi s_W^3c_W^3}+\frac{\alpha v_u}{12\pi s_Wc_W}]U_Z(q^2)-\frac{\alpha c_W}{2\pi s_W}U_W(q^2)\\&+\frac{\alpha c_W}{\pi s_W}ln[\frac{1+cos\theta}{2}]ln\frac{q^2}{M_W^2}+\frac{\alpha v_u(1-v_e^2)}{32\pi s_W^3c_W^3}ln[\frac{1+cos\theta}{1-cos\theta}]ln\frac{q^2}{M_W^2}\\&-\frac{\alpha c_W}{12\pi s_W}\ln\frac{q^2}{m_t^2}\left(\frac{m_t^2}{M_W^2}-\frac{m_b^2}{M_W^2}\right)\end{aligned} \quad (2.9)$$

$$\begin{aligned}V^S_{Z\gamma,lu}(q^2,\theta)=&\frac{\alpha(3-2s_W^2)}{24\pi s_Wc_W}U_W(q^2)-[\frac{3\alpha v_u(1-v_u^2)}{256\pi s_W^3c_W^3}+\frac{\alpha v_e}{8\pi s_Wc_W}]U_Z(q^2)-\frac{\alpha c_W}{2\pi s_W}U_W(q^2)\\&+\frac{\alpha c_W}{\pi s_W}ln[\frac{1+cos\theta}{2}]ln\frac{q^2}{M_W^2}+\frac{3\alpha v_e(1-v_u^2)}{64\pi s_W^3c_W^3}ln[\frac{1+cos\theta}{1-cos\theta}]ln\frac{q^2}{M_W^2}\\&-\frac{\alpha}{16\pi s_Wc_W}\ln\frac{q^2}{m_t^2}\left(1-\frac{4s_W^2}{3}\right)(\frac{m_t^2}{M_W^2}-\frac{m_b^2}{M_W^2})\end{aligned} \quad (2.10)$$

$$\begin{aligned}\tilde{\Delta}^S_{\alpha,ld}(q^2,\theta)=&-\frac{\alpha}{6\pi}U_W(q^2)+\frac{\alpha(2-v_e^2-v_d^2)}{64\pi s_W^2c_W^2}U_Z(q^2)+\frac{\alpha}{2\pi}U_W(q^2)\\&-\frac{\alpha}{\pi}ln[\frac{1-cos\theta}{2}]ln\frac{q^2}{M_W^2}+\frac{3\alpha(1-v_e^2)(1-v_d^2)}{256\pi s_W^4c_W^4}ln[\frac{1+cos\theta}{1-cos\theta}]ln\frac{q^2}{M_Z^2}\\&-\frac{\alpha}{24\pi s_W^2}(ln\frac{q^2}{m_t^2})\ [\ s_W^2(\frac{m_t^2}{M_W^2})+(3-s_W^2)(\frac{m_b^2}{M_W^2})\ ]\end{aligned} \quad (2.11)$$

$$R^S_{ld}(q^2,\theta)=\frac{\alpha}{4\pi s_W^2}(1-\frac{2s_W^2}{3})U_W(q^2)-\frac{\alpha(2+3v_e^2+3v_d^2)}{64\pi s_W^2c_W^2}U_Z(q^2)-\frac{\alpha c_W^2}{2\pi s_W^2}U_W(q^2)$$

$$+\frac{\alpha c_W^2}{\pi s_W^2} ln[\frac{1-cos\theta}{2}] ln\frac{q^2}{M_W^2} - \frac{\alpha v_e v_d}{4\pi s_W^2 c_W^2} ln[\frac{1+cos\theta}{1-cos\theta}] ln\frac{q^2}{M_Z^2}$$
$$+\frac{\alpha}{16\pi s_W^2}(ln\frac{q^2}{m_t^2})\ [(1-\frac{2s_W^2}{3})(\frac{m_t^2}{M_W^2}) + (1+\frac{2s_W^2}{3})(\frac{m_b^2}{M_W^2})\ ] \tag{2.12}$$

$$V^S_{\gamma Z,ld}(q^2,\theta) = \frac{\alpha(3+4c_W^2)}{24\pi s_W c_W} U_W(q^2) - [\frac{\alpha v_e(1-v_e^2)}{128\pi s_W^3 c_W^3} + \frac{\alpha v_d}{24\pi s_W c_W}] U_Z(q^2) - \frac{\alpha c_W}{2\pi s_W} U_W(q^2)$$
$$+\frac{\alpha c_W}{\pi s_W} ln[\frac{1-cos\theta}{2}] ln\frac{q^2}{M_W^2} - \frac{\alpha v_u(1-v_e^2)}{32\pi s_W^3 c_W^3} ln[\frac{1+cos\theta}{1-cos\theta}] ln\frac{q^2}{M_W^2}$$
$$+\frac{\alpha c_W}{24\pi s_W}(ln\frac{q^2}{m_t^2})\ [\ (\frac{m_t^2}{M_W^2}) - (\frac{m_b^2}{M_W^2})\ ] \tag{2.13}$$

$$V^S_{Z\gamma,ld}(q^2,\theta) = \frac{\alpha(3-4s_W^2)}{24\pi s_W c_W} U_W(q^2) - [\frac{3\alpha v_d(1-v_d^2)}{128\pi s_W^3 c_W^3} + \frac{\alpha v_e}{8\pi s_W c_W}] U_Z(q^2) - \frac{\alpha c_W}{2\pi s_W} U_W(q^2)$$
$$+\frac{\alpha c_W}{\pi s_W} ln[\frac{1-cos\theta}{2}] ln\frac{q^2}{M_W^2} - \frac{3\alpha v_e(1-v_d^2)}{32\pi s_W^3 c_W^3} ln[\frac{1+cos\theta}{1-cos\theta}] ln\frac{q^2}{M_W^2}$$
$$-\frac{\alpha}{16\pi s_W c_W}(ln\frac{q^2}{m_t^2})(1-\frac{2s_W^2}{3})[(\frac{m_t^2}{M_W^2}) - (\frac{m_b^2}{M_W^2})\ ] \tag{2.14}$$

where $v_f \equiv 1 - 4s_W^2|Q_f|$.

Before moving to a detailed numerical investigation of the Sudakov effect on the various experimental observables of the process, there are a few preliminary general remarks that, we feel, might be relevant. In particular, the following points should be mentioned:

I) The universal and the non universal sets, that we have grouped in the various equations, should be in our opinion, <u>separately</u> gauge-independent. Given the fact that for all the <u>overall</u> listed form factors, by construction, this property holds true, the same feature must obtain both for the <u>overall</u> contributions of <u>non universal</u> kind and for the <u>overall</u> contributions of <u>universal</u> kind that are considered. Gauge dependence can only affect, separately, the <u>universal</u> contributions coming from the $2W$ vertex and from the W box. But their special combination, that builds the same universal contribution $U_W(q^2)$ produced by the (<u>gauge-independent</u>) single W vertex, must necessarily be gauge-independent as well. This fact reproduces an analogous well known property [17] of the $2W$ vertex. In fact, the so-called "pinch" component [18] of this vertex is gauge-dependent, and combines with a corresponding gauge-dependent part in the various (γ, Z) self-energies to make up gauge-independent quantities that Degrassi and Sirlin call "gauge-independent" self-energies, that produce the correct asymptotic RG logarithmic behaviour of the running couplings. In our case, a combination of the $2W$ vertex with a <u>box</u> generates the "correct" gauge-independent asymptotic Sudakov logarithmic behaviour (that exponentiates).

II) From the practical point of view of the validity of a perturbative expansion truncated at one loop, we believe that one should consider the terms in the various brackets and discuss the various effects separately in all observables. For instance, a cancellation might arise between the θ-independent and the θ-dependent contributions if they were both large and of opposite sign. This, we believe, would not make a one loop approximation reliable.

III) As a rather academic feature, we believe that it should be stressed that, for all "light" (massless) fermion production processes, there exists a "Magic Energy" where the θ-independent functions $U_W(q^2)$, $U_Z(q^2)$ both vanish. This corresponds to the choice

$$ln\frac{q^2}{M_W^2} \simeq ln\frac{q^2}{M_Z^2} = 3 \tag{2.15}$$

that selects the "Magic Energy"

$$\sqrt{q^2} \simeq 360 \text{ GeV} \tag{2.16}$$

Clearly, in the vicinity of this energy, <u>all</u> the logarithmic Sudakov contribution for massless fermions is produced by the θ-dependent, non universal components of the weak boxes (which reduces essentially, from the numerical point of view, to the contribution from the W box). For bottom production at this energy, an extra amount of Yukawa Sudakov logarithms must be added (for top production, we believe that 360 GeV is definitely not an "asymptotic" energy, and the validity of an asymptotic expansion is strongly debatable; we shall only treat top production in this paper in the CLIC $\sqrt{q^2} = 3$ TeV regime).

After these general remarks, we are now ready to perform a numerical investigation of the various asymptotic Sudakov logarithms effects on all the observables of the process. This will be done in the next Section 3.

III. EFFECTS OF THE DIFFERENT SUDAKOV LOGARITHMS ON THE EXPERIMENTAL OBSERVABLES

Having examined the way in which the θ-independent Sudakov logarithms always group, for final massless fermions, in the combination $(3lnq^2 - ln^2q^2)$, for which precise rules exist [13] that make its resummation known (these are also available for the massive Yukawa contributions), we shall now proceed to the calculation of the Sudakov effects on various observables at one loop. With this aim, we shall consider the effects as due to three separate categories of terms : those which arise from θ-independent quantities and enter in the two possible combinations $U_W(q^2)$, $U_Z(q^2)$ eqs.(2.2,2.3); those which arise from θ-dependent terms (denoted "θ S"), and those which are of massive Yukawa origin (denoted "YU"). Since we are only interested in these specific contributions, we shall only write their (relative) effects on the different cross sections and their (absolute) effects on the different asymmetries, denoting with a "NS" ($\equiv$ Non Sudakov) symbol all the remaining part of the various observables. For sake of comparison, we shall also include in our formulae the relative and absolute effects due to the linear logarithms of Renormalization Group (RG) origin, already computed (e.g. [7]). Also, for simplicity, we shall group together the two W and Z functions writing $U_W \simeq U_Z \simeq U$ ($M_W \simeq M_Z \equiv M$), which creates a small numerical difference that will be irrelevant for the specific purposes of this paper, given the fact that the Z term is much smaller than the W one. We shall consider as realistic observable final states those which contain a $\mu^+\mu^-$ (or , also, a $\tau^+\tau^-$ pair), a $b\bar{b}$ pair, a $t\bar{t}$ pair. Also, the cross section for production of the five "light" quarks σ_5 will be considered. Note that for what concerns top production our formalism, strictly speaking, only applies to energies in the CLIC range [8], and for this reason this process will not be studied in the LC regime. We shall

restrict our attention on cross sections for production of a single final state $f\bar{f}$ (σ_f), on σ_5, and on forward-backward asymmetries ($A_{FB,f}$) and also longitudinal polarization asymmetries ($A_{LR,f}$) whose conventional definitions are recalled in Appendix B. Starting from the expressions given in Appendix B and from Eqs.(2.4-2.14) it is a relatively straightforward task to derive the various Sudakov effects. We shall write them in what follows replacing the theoretical input weak parameters by their experimental values, to make the different numerical size of the various terms immediately evident.

We now list the final expressions for the various observables. They read:

$$\sigma_\mu = \sigma_\mu^{NS}\{1 + \frac{\alpha(M)}{\pi}[(0.645\ ln\frac{q^2}{M^2})_{RG} + (1.51\ U(q^2)) + (5.49\ ln\frac{q^2}{M^2})_{\theta S}] \tag{3.1}$$

$$A_{FB,\mu} = A_{FB,\mu}^{NS} + \frac{\alpha(M)}{\pi}[(-1.07\ ln\frac{q^2}{M^2})_{RG} + (0.021\ U(q^2)) + (2.80\ ln\frac{q^2}{M^2})_{\theta S}] \tag{3.2}$$

$$A_{LR,\mu} = A_{LR,\mu}^{NS} + \frac{\alpha(M)}{\pi}[(-3.58\ ln\frac{q^2}{M^2})_{RG} + (0.92\ U(q^2)) + (5.13\ ln\frac{q^2}{M^2})_{\theta S}] \tag{3.3}$$

$$\begin{aligned}\sigma_b = \sigma_b^{NS}\{1 + \frac{\alpha(M)}{\pi}[(-5.30\ ln\frac{q^2}{M^2})_{RG} + (2.39\ U(q^2))\\ +(15.01\ ln\frac{q^2}{M^2})_{\theta S} - (2.10\ ln\frac{q^2}{m_t^2})_{YU}]\end{aligned} \tag{3.4}$$

$$\begin{aligned}A_{FB,b} = A_{FB,b}^{NS} + \frac{\alpha(M)}{\pi}[(-1.11\ ln\frac{q^2}{M^2})_{RG} + (0.098\ U(q^2))\\ +(4.25\ ln\frac{q^2}{M^2})_{\theta S} - (0.09\ ln\frac{q^2}{m_t^2})_{YU}]\end{aligned} \tag{3.5}$$

$$\begin{aligned}A_{LR,b} = A_{LR,b}^{NS} + \frac{\alpha(M)}{\pi}[(-3.71\ ln\frac{q^2}{M^2})_{RG} + (0.72U\ (q^2))\\ +(5.30\ ln\frac{q^2}{M^2})_{\theta S} - (0.60\ ln\frac{q^2}{m_t^2})_{YU}]\end{aligned} \tag{3.6}$$

$$\begin{aligned}\sigma_5 = \sigma_5^{NS}\{1 + \frac{\alpha(M)}{\pi}[(-3.26\ ln\frac{q^2}{M^2})_{RG} + (2.08\ U(q^2))\\ +(7.22\ ln\frac{q^2}{M^2})_{\theta S} - (0.30\ ln\frac{q^2}{m_t^2})_{YU}]\end{aligned} \tag{3.7}$$

$$\begin{aligned}R_b \equiv \frac{\sigma_b}{\sigma_5} = R_b^{NS} + \frac{\alpha(M)}{\pi}[(-0.29\ ln\frac{q^2}{M^2})_{RG} + (0.045\ U(q^2))\\ +(1.12\ ln\frac{q^2}{M^2})_{\theta S} - (0.26\ ln\frac{q^2}{m_t^2})_{YU}]\end{aligned} \tag{3.8}$$

$$A_{LR,5} = A^{NS}_{LR,5} + \frac{\alpha(M)}{\pi}[(-4.16\ ln\frac{q^2}{M^2})_{RG} + (0.92\ U(q^2))$$
$$+(3.67\ ln\frac{q^2}{M^2})_{\theta S} - (0.13\ ln\frac{q^2}{m_t^2})_{YU}] \tag{3.9}$$

$$\sigma_t = \sigma_t^{NS}\{1 + \frac{\alpha(M)}{\pi}[(-1.64\ ln\frac{q^2}{M^2})_{RG} + (1.80\ U(q^2))$$
$$+(1.18\ ln\frac{q^2}{M^2})_{\theta S} - (3.55\ ln\frac{q^2}{m_t^2})_{YU}] \tag{3.10}$$

$$A_{FB,t} = A^{NS}_{FB,t} + \frac{\alpha(M)}{\pi}[(-0.88\ ln\frac{q^2}{M^2})_{RG} + (0.06\ U(q^2))$$
$$-(0.93\ ln\frac{q^2}{M^2})_{\theta S} + (0.15\ ln\frac{q^2}{m_t^2})_{YU}] \tag{3.11}$$

$$A_{LR,t} = A^{NS}_{LR,t} + \frac{\alpha(M)}{\pi}[(-4.06\ ln\frac{q^2}{M^2})_{RG} + (1.01\ U(q^2))$$
$$+(0.76\ ln\frac{q^2}{M^2})_{\theta S} + (0.95\ ln\frac{q^2}{m_t^2})_{YU}] \tag{3.12}$$

In the case of top production, other observables can be added that depend on the final top helicity. They are listed in the second of Ref. [8]. As it was already remarked in that reference, these extra observables only differ from their corresponding "unpolarized top" quantities by linear Sudakov logarithms of θ-dependent box origin. Therefore the conclusions concerning the θ-independent terms will remain unchanged. A more complete discussion about the θ-dependent effects could be given, but it seems to us beyond the specific purposes of this paper. We shall defer to a dedicated forthcoming paper devoted to top production [19] for more details.

We may try to compare the above expressions with results obtained by other authors e.g [9,12,13]. However the comparison is not obvious because the photonic part is treated differently in these papers and the observables are often not defined in the same way. One can nevertheless identify the main terms. The comparison is easier with the results in ref [12]. They differ only by the inclusion of the small θ-dependent contribution from the γZ boxes and are indeed very close to our results, as one can easily verify.

We are now ready for a detailed numerical investigation of the Sudakov effects on the listed observables at one loop. With this aim, we divide our analysis into two parts, separately devoted to the two cases of energies in the LC($\sqrt{q^2} \simeq 500$ GeV) and in the CLIC($\sqrt{q^2} \simeq 3$ TeV) regime. Our main conclusions can be summarized as follows:

I) LC regime ($\sqrt{q^2} \simeq 500$ GeV)

At $\sqrt{q^2} \simeq 500$ GeV, the Sudakov effects act on the various observables in quite a different way. We have made the following general classification:

a) Cross sections. The relative effect in permille on the muon cross section is, to good approximation, a negative five from the θ-independent and a positive forty-six from the θ-dependent term. For the "light" quark cross section σ_5 it is a negative seven (θ-independent) and a positive sixty-one (θ-dependent). For bottom production, the relative effect on the cross section is a negative eight (θ-independent) and a positive hundred-twentysix (θ-dependent). One has also, in this case, a negative relative effect of eleven permille coming from the extra linear Sudakov logarithm of Yukawa origin.

The general comment that can be made at this point is that, for all the considered cross sections, the effect of the non universal, θ-dependent subleading Sudakov logarithm is, at one loop, by far larger than that of the θ-independent combinations $U_W(q^2)$, $U_Z(q^2)$, and systematically of opposite sign. This is, somehow, unfortunate since a resummation prescription for the θ-dependent logarithms does not seem to exist at the moment [13]. In the LC range, this might not represent a problem for a one loop approximation if one considers the relative one percent as a reasonable experimental achievement for σ_μ and σ_5. In this case, relative effects at one loop around five percent might be tolerated, with some warning in the case of σ_5. For bottom production, if one expect an experimental accuracy of a few percent, a thirteen percent effect would still be acceptable. If the experimental precision were higher than the previous qualitative estimates given here, the necessity of a two loop calculation for the θ-dependent contribution would become imperative. Note, accidentally, that the effects of the resummable terms $U_W(q^2)$, $U_Z(q^2)$, are in this case extremely small at the one loop level, so that in their case a one loop approximation seem to us completely reliable.

b) Longitudinal polarization asymmetries. This case presents strong similarities with that of the corresponding cross sections, and therefore we treat it immediately in succession. The absolute effect (in permille) on the muon asymmetry, $A_{LR,\mu}$ is a negative three (θ-independent) and a positive forty-three (θ-dependent). For the five light quarks case $A_{LR,5}$, the absolute effect is a negative three (θ-independent) and a positive thirty-one (θ-dependent). For bottom production, $A_{LR,b}$, the two effects are respectively minus two and plus forty-five. Again, one notices a strong dominance at one loop of the positive θ-dependent terms with respect to the negative θ-independent ones, just as in the case of cross sections. The reliability of the one loop approximation will strongly depend on the aimed experimental accuracies of the measurements. If these will remain at the (absolute) few percent level, there should be no problem for the approximation, while higher experimental accuracies would make a two-loop calculation of the θ-dependent terms highly "desirable".

c) Forward-backward asymmetries. These specific observables present a peculiar feature that extremizes the previously remarked "θ-dependent logarithms dominance". In fact, in their case, an accurate numerical calculation shows that, independently of the considered final state, the coefficient of the θ-independent $\simeq U_W(q^2)$, $U_Z(q^2)$ terms is always, essentially, negligibly small, i.e. much smaller than that of the θ-dependent term and well below the absolute percent level. In Appendix A we have tried to derive in some detail this apparently non trivial fact, which seems to arise from the multiplets assignment of the fermions in $SU(2)\times U(1)$. Numerically and to a good approximation this absolute effect is (in permille) always negative and in magnitude well less than one for fi-

nal muons and one for $b's$. This feature will persist at the higher energies involved at CLIC, where it will apply also to top production (that we do not treat at LC energies), and seems to be a very general property of this type of observables. The consequence is that, at asymptotic energies, the only Sudakov logarithms that must be retained at one loop in the forward-backward asymmetries are the θ-dependent ones of box origin. This generates a strange situation of "box dominance" for what concerns this type of virtual effects, totally opposite to the situation e.g. met on top of the Z resonance.[1]

For what concerns the validity of a one-loop approximation, the θ-dependent absolute effects are always positive and equal, in permille, to twenty-four (final muons) and thirty-six (final bottom). At the percent level of experimental accuracies, this seems to us to make the approximation reliable, even in cases of reasonable improvements in the experimental precision.

II) CLIC regime ($\sqrt{q^2} = 3$ TeV)

For energies of about 3 TeV, as those aimed for in the first phase of the future CERN CLIC Collider, we have repeated the previous analysis including also top production for which CLIC energies can be safely considered as "asymptotic". We list here the results that we have found, quoting the θ-independent term first, then giving the θ-dependent term effect and finally, whenever involved, giving the Yukawa term contribution (in percent this time, relative for cross sections and absolute for asymmetries).

a) Cross sections. For final muons, we get a negative ten and a positive nine (percent). For final light quarks (σ_5), there appear a negative fourteen and a positive twelve. For bottom production, a negative sixteen and a positive twentysix (plus a negative three of Yukawa origin). For top production, a negative twelve and a positive two (with a negative five of Yukawa origin).

As one sees, the situation at CLIC is strongly different from the corresponding one at LC. The role of the θ-independent terms is now slightly more relevant than that of the θ-dependent ones for both muon and for light quarks production and largely dominant for top production, it remains less relevant only for bottom production.

For what concerns the validity of a one-loop approximation, the situation seems to us to be, in a certain sense, embarassing, and also final state dependent. For muon and light quark production, one might take the pragmatic attitude of considering the overall Sudakov effect, obtained by summing the negative θ-independent and the positive θ-dependent one. This sum is actually small (a few percent) and apparently under control. A more cautious point of view though, that we personally share, is that one is dealing here with two large

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[1] As we already anticipated in Eqs.(2.15,2.16), a similar and rather peculiar feature of the Sudakov logarithms arise at $\sqrt{q^2} \simeq 360$ GeV $\simeq 4\ M_Z$. Here the θ-independent term vanishes exactly at one loop, so that the full effect is produced, at that energy, by weak boxes. This situation is again just opposite to that met at the Z peak where boxes could be safely ignored, and affects not only the forward-backward asymmetries but all observables at this special energy value.

and opposite effects, that are both separately gauge-independent and of rather different origin, the positive one being completely non-universal and θ-dependent. We do not see any obvious reason why the two large and independent effects should still cancel e.g. at the next two-loop level. Thus, in our opinion, one should compute them both to higher order. This would not represent a problem for the θ-independent contribution, for which resummation prescriptions exist [13]. But, as we already said, these prescriptions are unclear for the θ-dependent term. Given its rather large size, a calculation of this quantity at the next two-loop level seems to us, least to say, unavoidable.

Note that, in our opinion, until a two-loop calculation of the latter term has been performed, resumming the θ-independent effect only, leaving the other terms at the one loop level, could worsen the situation. This procedure might in fact reduce the resummed negative contribution, leaving a much larger positive dominant term. Unfortunately, it seems to us that for light fermion production cross sections at CLIC energies, a resummation of the pure θ-independent terms, although theoretically valid and remarkable, does not provide the full answer to the need of a reliable, complete theoretical prediction.

This conclusion remains unchanged, in our opinion, also for bottom production, from inspection of the numerical effects that we have shown. The only apparent evasion of this negative statement is provided by the cross section for top production. Here the θ-independent effect dominates, while the other one is small and limited (two percent). The reason for the weakness of the θ-dependent term is the fact that its leading contribution, the W box diagram for $t\bar{t}$ production, has an angular distribution $\simeq ln[(1+cos\theta)/2]$ which is peaked backward (as one can guess from the diagram (2) of Fig.1d) and interfers very little with the forward peaked Born term; in the case of $b\bar{b}$ production the diagram (1) of Fig.1d, on the contrary, is peaked forward and interfers strongly with the Born term. For $t\bar{t}$ production in this situation, one could safely approximate the cross section with the one-loop calculation, resumming only the θ-independent term. This process could therefore be already satisfactorily calculated, without extra theoretical efforts, by a suitable combination of different existing formulae for the θ-independent [13] and the θ-dependent [8] contributions.

b) Polarization asymmetries.

One finds again, as in the LC case, a situation that is similar to that of the corresponding cross sections. The absolute numbers (in percent) are for final muons, $A_{LR,\mu}$, minus six and plus nine; for light quarks, $A_{LR,5}$, minus six and plus six; for final bottom, minus five and plus nine; for final top, minus seven and plus one. Assuming a (approximately) percent level for the related experimental precisions, we believe that the same conclusions, that were just drawn in (IIa) for what concerns the one-loop approximation at CLIC energies for the various light fermions, bottom, top cross sections, still apply for all the corresponding longitudinal polarization asymmetries.

c) Forward-backward asymmetries.

At CLIC energies, the absolute overall contribution of the θ-independent Sudakov terms is, for both massless and massive fermion production, systematically irrelevant (one permille for muons, few permille for either light or massive quarks) at the level of realistic expectable experimental accuracy. This is in agreement with our general previous observation, that will

be discussed separately in Appendix A. The box θ-dependent absolute effects are, respectively, five percent (final muons), seven percent (bottom production), minus two percent (top production). With an experimental accuracy of one percent for muons and top, and of a few percent for bottom production, a one-loop approximation seems to us fully acceptable. In this case, the relevant effect would be fully provided by existing one-loop calculations [7,8] of the angular dependent component of the terms, without need of any extra theoretical effort.

We have thus completed our numerical analysis in the two (LC, CLIC) different considered "asymptotic" energies. The main results and conclusions are summarized in the forthcoming and final Section 4.

IV. CONCLUSIONS

We have performed in this paper a systematic analysis of the weak Sudakov logarithmic effects at one loop in the 't Hooft $\xi = 1$ gauge for a large class of experimental observables, in two different energy configurations that correspond to the regimes to be explored at the next linear colliders LC (500 GeV) and CLIC (3 TeV). We have divided the set of effects into two essentially different gauge-independent subsets. The first one is "angular independent", is universal and comes from vertices and boxes; the second one is "angular dependent", is not universal and only comes from boxes. The main motivation of our analysis was that of studying the specific effects of this last term, for which no clean resummation prescription beyond the one loop level seems to exist at the moment [13].

Our numerical analysis has been explicitely performed at two selected energies, 500 GeV and 3 TeV, but it can be easily repeated for any arbitrary value e.g. beyond 500 GeV or 3 TeV. This has been summarized pictorially in Figs.(2-11), where we have shown the various contributions from the angular independent and from the angular dependent term on all the chosen observables, when the energy varies. For sake of completeness we have also enclosed the universal linear logarithmic contribution of RG origin, which has been computed in previous references [7,8] and does not seem to become "dangerous" at the considered energies. From inspection of these Figures, one sees e.g. that our conclusions remain essentially valid when $\sqrt{q^2}$ ranges between $\simeq 3$ and $\simeq 5$ TeV, a possible larger CLIC range, or between 500 GeV and 1 TeV in the LC case.

Our general conclusion is that, for each considered observable, in both the considered energy configurations, the role of the angular dependent term is <u>always</u> essential. In the cases of cross sections and longitudinal polarization asymmetries, our analysis has led to very similar conclusions for the "corresponding" quantities (i.e. the cross section for production of a certain final state and the related longitudinal polarization asymmetry). In practice, at LC, the angular dependent logarithms are dominant but "small" i.e. "under control" at one-loop, assuming an experimental accuracy of the percent size. At CLIC, a strong cancellation appears at one loop between the "large" (i.e. $\gtrsim 10$ %) negative angular independent effect and the "large" positive angular dependent one. So both should not be taken, in our opinion, in the one-loop approximation, which does not represent a problem for the first term, but requires a new calculation at least at two-loops for the second one. An exception to this statement is provided by the cross section for top production, the only case that we found where the angular dependent effect turns out to be negligible.

A completely separate role is played by the forward-backward asymmetries. In these observables, independently of the considered "high" energy, the angular independent effect at one-loop is essentially vanishing, for reasons that seem to be accidental. Thus the angular dependent Sudakov logarithm remains, for these special quantities, the only relevant effect. Luckily, if one assumes realistic experimental accuracies, this effect appears to be under control both at LC and at CLIC energies, which would allow to avoid a hard two-loop calculation in all cases.

As a matter of fact, the need of a two-loop calculation of the angular dependent term is only appearing for calculations of cross sections (of which longitudinal polarization asymmetries are essentially a special case). The possibility that a simpler calculational approach can be found for these well defined cases is, at the moment, being investigated.

A numerical simplification has occured, in fact, in our approach and we want to discuss it now briefly. In the calculation of the size of the effect of the θ-independent Sudakov terms on the various observables we have fully retained the asymptotic expressions given in Eqs.(2.4)-(2.14). In the two limiting situations of forward and backward scattering, $\cos\theta \to \pm 1$, the asymptotic expressions formally diverge like a logarithm. Clearly, this would not be the case if we had used the complete expression, which would be necessary in the low q^2 range. For the specific purposes of this paper, where only θ-integrated quantities have been considered and estimates of effects are essentially indicative (i.e. aiming at identifying "dangerous" potential contributions), this approximation seems to us satisfactory in the large q^2 regime in which we are interested. First of all, a logarithmic singularity produces in any case a finite integrated quantity. In second place, one must remember that in a realistic experiment there is always a finite value of the scattering angle, $\theta = \theta_0$, below which no experimental observation is allowed. Starting from these considerations we have first recomputed the integration of the θ-dependent logarithms with a cut at $\pm\cos\theta_0$, and compared these values with those obtained performing the full integration that would correspond to $\theta_0 = 0$. More precisely, we have considered the two quantities which appear in the W box contribution to the cross sections:

$$I_1 = \int_{-\cos\theta_0}^{\cos\theta_0} d\cos\theta\ (1+\cos^2\theta) ln(1 \mp \cos\theta) \tag{4.1}$$

$$I_2 = \mp\int_{-\cos\theta_0}^{\cos\theta_0} d\cos\theta\ \ \cos\theta\ ln(1 \mp \cos\theta) \tag{4.2}$$

These can be computed analytically leading to the expressions

$$\begin{aligned} I_1 = & -\frac{8}{3}\cos\theta_0 - \frac{2}{9}\cos^3\theta_0 + \frac{1}{3}(-4 + 3\cos\theta_0 + \cos^3\theta_0)\ln(1-\cos\theta_0) + \\ & + \frac{1}{3}(4 + 3\cos\theta_0 + \cos^3\theta_0)\ln(1+\cos\theta_0) \end{aligned} \tag{4.3}$$

$$I_2 = \cos\theta_0 - \frac{1}{2}\sin^2\theta_0 \ln\frac{1+\cos\theta_0}{1-\cos\theta_0} \tag{4.4}$$

and for example, one finds $I_1 = -1.04,\ -0.90,\ -0.67,\ -0.44$, and $I_2 = 1,\ 0.91,\ 0.74,\ 0.54$ when $\theta_0 = 0,\ 10^0,\ 20^0,\ 30^0$, respectively. As one sees, the "cut" quantities, for values of θ_0

as large as $\simeq 20^0$, only differ from the complete integration by a relative $20-30\%$ difference, and will essentially reproduce its main features, so that this cut effect will be irrelevant for our conclusions. In this region, our "logarithmic approximation" should be satisfactory. In fact, in the expressions to be integrated we made the assumption

$$\theta >> \frac{M_W}{\sqrt{q^2}} \tag{4.5}$$

which ensures that the terms $ln^2(t/M_W^2)$, $ln^2(u/M_W^2)$ are large and can safely be estimated by neglecting their q^2-independent parts.

At CLIC energies, the r.h.s. of Eq.(4.5) is equal to $\simeq 1.5$ degree while in the LC range it reaches a value of about 9 degrees (or less, for $\sqrt{q^2} > 500$ GeV). In both cases, from what previously shown, an estimate of the angular cut that must be performed in the different observables, based on the logarithmic approximation truncated, say, at a corresponding realistic cut, would reproduce "essentially" the numbers that we gave, to a degree of accuracy that should be fixed by a dedicated analysis of the special experimental features of the related experiments.

We notice at this point that the same logarithmic approximation that we followed was also used in Ref. [12], where a detailed numerical analysis has been performed, with precise numbers, for some of the observables that we considered. Since the analysis of Ref. [12] also includes, as we already mentioned, the θ-dependent contribution from the γZ boxes, slight differences appear in the various results.

A final comment should now be made concerning the role played by other possible asymptotic logarithms in the examined processes. The (linear ones) of RG origin have been listed in our formulae, and the reader can very easily verify that their effect at one loop will never be "dangerous" at the considered energies. There is another interesting possibility, due to virtual Sudakov effects at one loop of supersymmetric origin. They have been exhaustively discussed for the MSSM case, in the case $f \neq t$ in the few TeV regime in a recent paper [20]. They turn out to be only linearly logarithmic, and systematically "under control" at CLIC energies with a possible exception for bottom production (assuming a typical SUSY mass of few hundred GeV). So, for $f \neq t$ production, SUSY does not add technical problems at the theoretical one-loop level in the MSSM case. Note that, also in the MSSM case, the extra SUSY θ-independent contributions to the considered forward-backward asymmetries are systematically negligible, exactly as in the SM case.

In the case $f = t$, discussed in the second of Ref. [8], the situation is slightly less straightforward. The size of the linear Sudakov logarithm, that contains a large component of Yukawa origin, depends strongly on the SUSY parameter $\tan\beta$. For the lowest allowed values of $\tan\beta$, its numerical value in the cross section can be larger than the ten percent "safety" limit. A strong reduction of the effect is, though, achievable by adding in the asymptotic expansion a reasonable extra SUSY constant [2]. A full and detailed discussion on this important point will appear soon in a forthcoming dedicated paper [19].

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[2] As a matter of fact, in Refs. [7,8] an asymptotic expansion at the one-loop level, in the TeV range, was used of the more complete form $aU(q^2)+b_{\theta S}(\log q^2/M^2)_{\theta S}+b_{RG}(\log q^2/M^2)_{RG}+c$. The numerical values of the constants c in the various cases turned out to correspond systematically to

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a negative few percent (relative or absolute) effect, with a and b in agreement with the theoretical Sudakov and RG values. For LC energies, this decreased in all cases the overall logarithmic one loop effect, that was already in our philosophy under control, being the sum of two separately small effects. ¿From this fact we would be led to reinforce the conclusion that for LC energies the complete one loop approximation should be satisfactory, at least at the percent experimental level of accuracy.

APPENDIX A: THE SPECIAL BEHAVIOUR OF THE FORWARD-BACKWARD ASYMMETRIES

In this short appendix we investigate the origin of the fact that the θ-independent contributions to $A_{FB,f}$ turn out to be small in the high energy limit ($s >> M^2$).

In the separate production of chiral fermions (f_L or f_R) a θ-independent correction $C^f_{L,R}$ does obviously not modify the Born values of $A_{FB,f_{L,R}}$ as it gives the same effect in the forward and in the backward domains. However, since $C^e_L \neq C^e_R$, $C^f_L \neq C^f_R$, and since the integrated Born cross sections in the forward or in the backward domains $\sigma^{Born}_{F,B}(e_{L,R}, f_{L,R})$ are non equal, it is apparently not obvious that the forward-backward asymmetry for unpolarized initial electrons and final fermions

$$
\begin{aligned}
A_{FB,f}= & \frac{\sigma_{F-B}(e_L,f_L)+\sigma_{F-B}(e_L,f_R)+\sigma_{F-B}(e_R,f_L)+\sigma_{F-B}(e_R,f_R)}{\sigma_{F+B}(e_L,f_L)+\sigma_{F+B}(e_L,f_R)+\sigma_{F+B}(e_R,f_L)+\sigma_{F+B}(e_R,f_R)} \\
& = [\sigma^{Born}_{F-B}(e_L,f_L)(1+C^e_L+C^f_L)+\sigma^{Born}_{F-B}(e_L,f_R)(1+C^e_L+C^f_R) \\
& +\sigma^{Born}_{F-B}(e_R,f_L)(1+C^e_R+C^f_L)+\sigma^{Born}_{F-B}(e_R,f_R)(1+C^e_R+C^f_R)]\times \\
& [\sigma^{Born}_{F+B}(e_L,f_L)(1+C^e_L+C^f_L)+\sigma^{Born}_{F+B}(e_L,f_R)(1+C^e_L+C^f_R) \\
& +\sigma^{Born}_{F+B}(e_R,f_L)(1+C^e_R+C^f_L)+\sigma^{Born}_{F+B}(e_R,f_R)(1+C^e_R+C^f_R)]^{-1}
\end{aligned}
\tag{A1}
$$

remains so close to its Born value.

The condition is that

$$
\begin{aligned}
& [C^f_L-C^f_R][\sigma^{Born}_F(e_L,f_L)(\sigma^{Born}_B(e_L,f_R)+\sigma^{Born}_B(e_R,f_R)) \\
& -\sigma^{Born}_B(e_L,f_L)(\sigma^{Born}_F(e_L,f_R)+\sigma^{Born}_F(e_R,f_R)) \\
& -\sigma^{Born}_F(e_L,f_R)\sigma^{Born}_B(e_R,f_L)+\sigma^{Born}_B(e_L,f_R)\sigma^{Born}_F(e_R,f_L) \\
& +\sigma^{Born}_F(e_R,f_L)\sigma^{Born}_B(e_R,f_R)-\sigma^{Born}_B(e_R,f_L)\sigma^{Born}_F(e_R,f_R)] \\
\simeq \quad & [C^e_R-C^e_L][\sigma^{Born}_F(e_L,f_L)(\sigma^{Born}_B(e_R,f_L)+\sigma^{Born}_B(e_R,f_R)) \\
& -\sigma^{Born}_B(e_L,f_L)(\sigma^{Born}_F(e_R,f_L)+\sigma^{Born}_F(e_R,f_R)) \\
& +\sigma^{Born}_F(e_L,f_R)\sigma^{Born}_B(e_R,f_L)-\sigma^{Born}_B(e_L,f_R)\sigma^{Born}_F(e_R,f_L) \\
& +\sigma^{Born}_F(e_L,f_R)\sigma^{Born}_B(e_R,f_R)-\sigma^{Born}_B(e_L,f_R)\sigma^{Born}_F(e_R,f_R)]
\end{aligned}
\tag{A2}
$$

and it is not trivially satisfied.

We have tried to analyse the contents of eq.(A2) and to look for the origin of the cancellations which appear in this expression or in the equivalent one, Eqs. (B10), given at the end of Appendix B, which can be used in the case of angular independent contributions:

$$A_{FB,f} = \frac{3}{4}\frac{U_{12}}{U_{11}} \tag{A3}$$

in which the photon and Z exchange terms are explicitely written.

Simplifications arise when one uses the fact that $s^2_W \simeq \frac{1}{4}$ which makes the vector coupling of the Z boson to l^+l^- ($l = e, \mu, \tau$) vanish (all the results obtained below would not be valid for an arbitrary value of s^2_W).

We first consider in the $s^2_W \simeq \frac{1}{4}$ approximation the process $e^+e^- \to \mu^+\mu^-$, where the photon Born term is purely vector and the Z Born term purely axial. One easily sees that

the one-loop corrections factorize out in the same way $(1 + 2C_L^l + 2C_R^l)$ in the numerator U_{12} (only given by the photon-Z interference) and in the denominator U_{11} (only given by the squared photon and the squared Z terms), so that their total effect in $A_{FB,f}$ vanishes. So in practice these θ-independent one-loop corrections should be proportional to $(1 - 4s_W^2)$ and indeed very small.

The case of the processes $e^+e^- \to u\bar{u}$ and $e^+e^- \to d\bar{d}$ is less obvious. One still uses the fact that the photon Born term is purely vector and that the initial Z Born coupling to e^+e^- is purely axial in the limit $s_W^2 = \frac{1}{4}$. In this limit, another essential ingredient is the numerical value of

$$\frac{4}{3}A_{FB,f}^{Born} = \left(\frac{U_{12}}{U_{11}}\right)^{Born} = \frac{3|Q_f|}{1 - |Q_f| + 5|Q_f|^2} \tag{A4}$$

which is close to 1 for both up and down quarks, i.e., $\frac{18}{23}$ for u and $\frac{9}{11}$ for d.

Including the angular independent corrections at first order leads to:

$$\frac{4}{3}A_{FB,f} = \left(\frac{3|Q_f|}{1 - |Q_f| + 5|Q_f|^2}\right)\left(\frac{1 + c_1^f + c_2^f}{1 + c_1^f + c_3^f}\right) \tag{A5}$$

where

$$c_1^f = C_L^l + C_R^l + C_L^f + C_R^f \tag{A6}$$

$$c_2^f = \left(\frac{1 - |Q_f|}{|Q_f|}\right)[|Q_f|(C_L^f - C_R^f) + \frac{1}{3}(C_L^l - C_R^l)] \tag{A7}$$

$$c_3^f = \left(\frac{3(1 - |Q_f|)}{(1 - |Q_f| + 5|Q_f|^2}\right)[|Q_f|(C_L^l - C_R^l) + \frac{1}{3}(C_L^f - C_R^f)] \tag{A8}$$

One sees now that, in addition to the term c_1^f which would factorize out like in the case $f = l$, there appear additional corrections c_2^f and c_3^f (which vanish for $f = l$). However these additional corrections turn out to be both of the same size, $c_2^f \simeq c_3^f$, and smaller than c_1^f in each of the cases $f = u$ and $f = d$. So at the end the total correction to the Born value is again rather small. One can trace the origin of the relation $c_2^f \simeq c_3^f << c_1^f$ in the fact that, using the notations of ref. [13] for the θ-independent terms, the Left-handed corrections $\simeq \frac{3}{4} + \frac{Y_L^2}{4}tan^2\theta_W$ are larger than the Right-handed ones $\simeq \frac{Q_f^2}{4}tan^2\theta_W$ (this is the usual electroweak feature), and also in the fact that $3|Q_f| \simeq 1 - |Q_f| + 5|Q_f|^2$ (leading to $\frac{4}{3}A_{FB,f}^{Born} \simeq 1$ for both $f = u, d$).

So in conclusion it appears that the angular independent electroweak corrections to $A_{FB,f}$ turn out to be small for "accidental" reasons related to the Left versus Right structure of the electroweak multiplets and to the value $s_W^2 \simeq \frac{1}{4}$. We do not see any deeper physical reason.

APPENDIX B: THE GENERAL FORM OF THE POLARIZED $e^+e^- \to f\bar{f}$ CROSS SECTION IN THE Z-PEAK SUBTRACTED REPRESENTATION

The general expression of the $e^+e^- \to f\bar{f}$ cross section can be written as

$$\frac{d\sigma_f}{dcos\theta}(P,P') = \frac{4\pi}{3}\mathcal{N}_f q^2\{\frac{3}{8}(1+cos^2\theta)[(1-PP')U_{11}+(P'-P)U_{21}]$$
$$+\frac{3}{4}cos\theta[(1-PP')U_{12}+(P'-P)U_{22}]\} \tag{B1}$$

where

$$U_{11} = \frac{\alpha^2(0)Q_f^2}{q^4}[1+2\tilde{\Delta}^{(lf)}_\alpha(q^2,\theta)]$$
$$+2[\alpha(0)|Q_f|]\frac{q^2-M_Z^2}{q^2((q^2-M_Z^2)^2+M_Z^2\Gamma_Z^2)}[\frac{3\Gamma_l}{M_Z}]^{1/2}[\frac{3\Gamma_f}{\mathcal{N}_f M_Z}]^{1/2}\frac{\tilde{v}_l\tilde{v}_f}{(1+\tilde{v}_l^2)^{1/2}(1+\tilde{v}_f^2)^{1/2}}$$
$$\times[1+\tilde{\Delta}^{(lf)}_\alpha(q^2,\theta)-R^{(lf)}(q^2,\theta)-4s_lc_l\{\frac{1}{\tilde{v}_l}V^{(lf)}_{\gamma Z}(q^2,\theta)+\frac{|Q_f|}{\tilde{v}_f}V^{(lf)}_{Z\gamma}(q^2,\theta)\}]$$
$$+\frac{[\frac{3\Gamma_l}{M_Z}][\frac{3\Gamma_f}{\mathcal{N}_f M_Z}]}{(q^2-M_Z^2)^2+M_Z^2\Gamma_Z^2}$$
$$\times[1-2R^{(lf)}(q^2,\theta)-8s_lc_l\{\frac{\tilde{v}_l}{1+\tilde{v}_l^2}V^{(lf)}_{\gamma Z}(q^2,\theta)+\frac{\tilde{v}_f|Q_f|}{(1+\tilde{v}_f^2)}V^{(lf)}_{Z\gamma}(q^2,\theta)\}] \tag{B2}$$

$$U_{12} = 2[\alpha(0)|Q_f|]\frac{q^2-M_Z^2}{q^2((q^2-M_Z^2)^2+M_Z^2\Gamma_Z^2)}[\frac{3\Gamma_l}{M_Z}]^{1/2}[\frac{3\Gamma_f}{\mathcal{N}_f M_Z}]^{1/2}\frac{1}{(1+\tilde{v}_l^2)^{1/2}(1+\tilde{v}_f^2)^{1/2}}$$
$$\times[1+\tilde{\Delta}^{(lf)}_\alpha(q^2,\theta)-R^{(lf)}(q^2,\theta)]$$
$$+\frac{[\frac{3\Gamma_l}{M_Z}][\frac{3\Gamma_f}{\mathcal{N}_f M_Z}]}{(q^2-M_Z^2)^2+M_Z^2\Gamma_Z^2}[\frac{4\tilde{v}_l\tilde{v}_f}{(1+\tilde{v}_l^2)(1+\tilde{v}_f^2)}]$$
$$\times[1-2R^{(lf)}(q^2,\theta)-4s_lc_l\{\frac{1}{\tilde{v}_l}V^{(lf)}_{\gamma Z}(q^2,\theta)+\frac{|Q_f|}{\tilde{v}_f}V^{(lf)}_{Z\gamma}(q^2,\theta)\}] \tag{B3}$$

$$U_{21} = 2[\alpha(0)|Q_f|]\frac{q^2-M_Z^2}{q^2((q^2-M_Z^2)^2+M_Z^2\Gamma_Z^2)}[\frac{3\Gamma_l}{M_Z}]^{1/2}[\frac{3\Gamma_f}{\mathcal{N}_f M_Z}]^{1/2}\frac{\tilde{v}_f}{(1+\tilde{v}_l^2)^{1/2}(1+\tilde{v}_f^2)^{1/2}}$$
$$\times[1+\tilde{\Delta}^{(lf)}_\alpha(q^2,\theta)-R^{(lf)}(q^2,\theta)-\frac{4s_lc_l|Q_f|}{\tilde{v}_f}V^{(lf)}_{Z\gamma}(q^2,\theta)]$$
$$+\frac{[\frac{3\Gamma_l}{M_Z}][\frac{3\Gamma_f}{\mathcal{N}_f M_Z}]}{(q^2-M_Z^2)^2+M_Z^2\Gamma_Z^2}[\frac{2\tilde{v}_l}{(1+\tilde{v}_l^2)}]$$
$$\times[1-2R^{(lf)}(q^2,\theta)-4s_lc_l\{\frac{1}{\tilde{v}_l}V^{(lf)}_{\gamma Z}(q^2,\theta)+\frac{2\tilde{v}_f|Q_f|}{(1+\tilde{v}_f^2)}V^{(lf)}_{Z\gamma}(q^2,\theta)\}] \tag{B4}$$

$$U_{22} = 2[\alpha(0)|Q_f|]\frac{q^2-M_Z^2}{q^2((q^2-M_Z^2)^2+M_Z^2\Gamma_Z^2)}[\frac{3\Gamma_l}{M_Z}]^{1/2}[\frac{3\Gamma_f}{\mathcal{N}_f M_Z}]^{1/2}\frac{\tilde{v}_l}{(1+\tilde{v}_l^2)^{1/2}(1+\tilde{v}_f^2)^{1/2}}$$

$$\times[1+\tilde{\Delta}^{(lf)}_{\alpha}(q^2,\theta)-R^{(lf)}(q^2,\theta)-\frac{4s_lc_l}{\tilde{v}_l}V^{(lf)}_{\gamma Z}(q^2)]$$

$$+\frac{[\frac{3\Gamma_l}{M_Z}][\frac{3\Gamma_f}{\mathcal{N}_f M_Z}]}{(q^2-M^2_Z)^2+M^2_Z\Gamma^2_Z}[\frac{2\tilde{v}_f}{(1+\tilde{v}^2_f)}]$$

$$\times[1-2R^{(lf)}(q^2,\theta)-4s_lc_l\{\frac{2\tilde{v}_l}{(1+\tilde{v}^2_l)}V^{(lf)}_{\gamma Z}(q^2,\theta)+\frac{|Q_f|}{\tilde{v}_f}V^{(lf)}_{Z\gamma}(q^2,\theta)\}] \tag{B5}$$

Here P, P' are the longitudinal polarization degree of the initial lepton and antilepton, and $\mathcal{N}_f$ is the colour factor for the $f\bar{f}$ channel which includes the appropriate QCD corrections to the input.

¿From this general expression one obtains the unpolarized integrated cross section

$$\sigma_f=\int^{+1}_{-1}dcos\theta\ \frac{d\sigma_f}{dcos\theta}(0,0) \tag{B6}$$

the forward backward asymmetry

$$A_{FB,f}=\left(\int^{+1}_{0}dcos\theta\ \frac{d\sigma_f}{dcos\theta}(0,0)-\int^{0}_{-1}dcos\theta\ \frac{d\sigma_f}{dcos\theta}(0,0)\right)/\sigma_f \tag{B7}$$

and the longitudinal polarization asymmetry

$$A_{LR,f}=\left(\int^{+1}_{-1}dcos\theta\ \frac{d\sigma_f}{dcos\theta}(-1,0)-\int^{+1}_{-1}dcos\theta\ \frac{d\sigma_f}{dcos\theta}(+1,0)\right)/2\sigma_f \tag{B8}$$

Note that for θ-independent contributions these integrals simplify and allow to write

$$\sigma_f=\frac{4\pi}{3}\mathcal{N}_f q^2 U_{11} \tag{B9}$$

$$A_{FB,f}=\frac{3}{4}\frac{U_{12}}{U_{11}} \tag{B10}$$

$$A_{LR,f}=\frac{U_{21}}{U_{11}}\ . \tag{B11}$$

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FIGURES

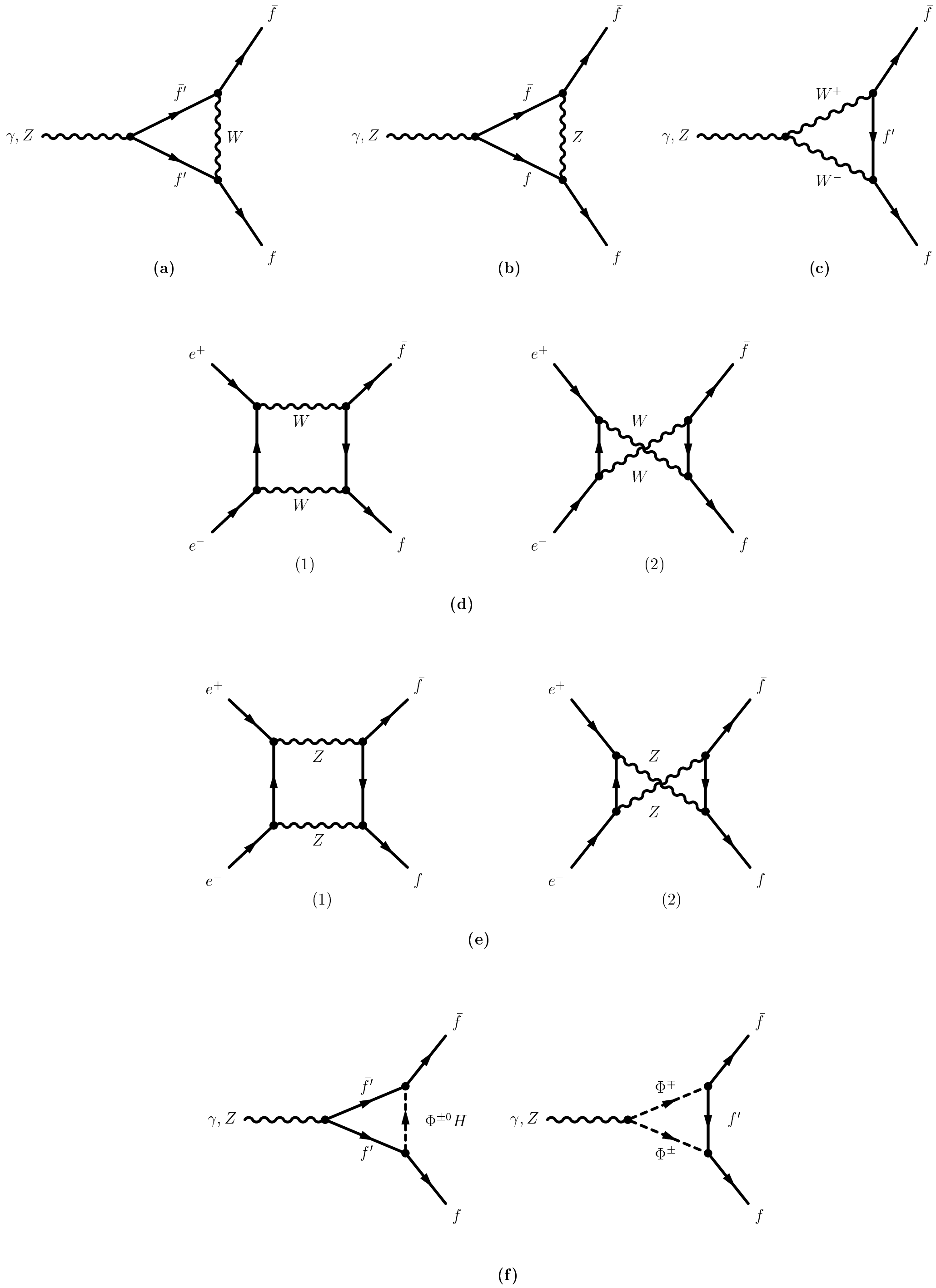

FIG. 1. SM diagrams contributing in the asymptotic regime. Note that for the WW box contributions (d), diagram (1) contributes for $I_{3f} = -\frac{1}{2}$, whereas diagram (2) contributes for $I_{3f} = +\frac{1}{2}$; for the ZZ box contributions (e), both diagrams contribute.

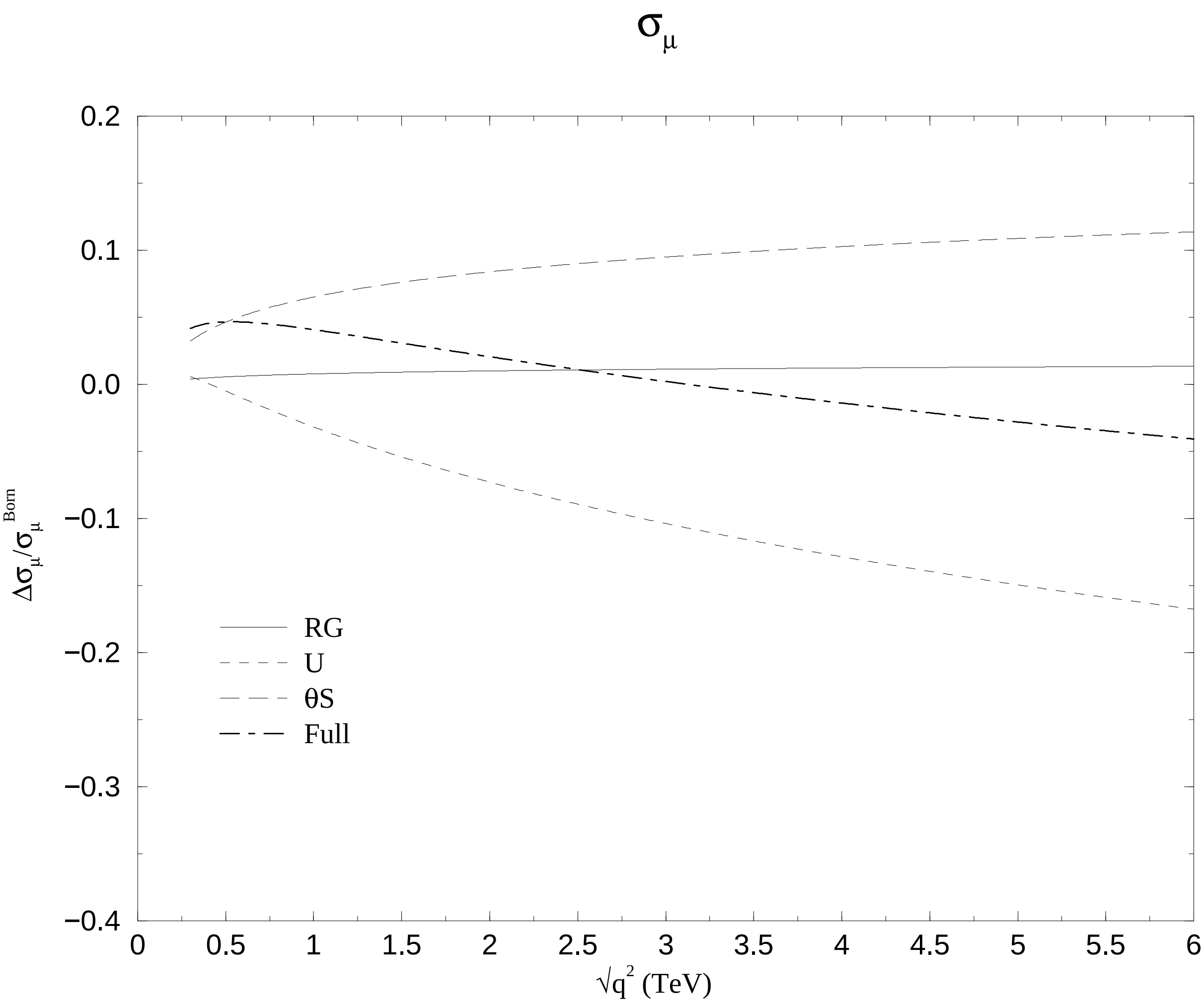

FIG. 2. Separate asymptotic contributions to σ_μ as functions of the energy. The solid line (RG) is the linear Renormalization Group logarithm. The dotted line (U) is the θ independent term proportional to the combination $3\log\frac{q^2}{M_Z^2} - \log^2\frac{q^2}{M_Z^2}$. The dashed line ($\theta S$) is the angular dependent linear logarithm. Finally, the thick dot-dashed line (Full) is the sum of the three contributions. The same captions applies to all the following figures showing the effects in all the other considered observables.

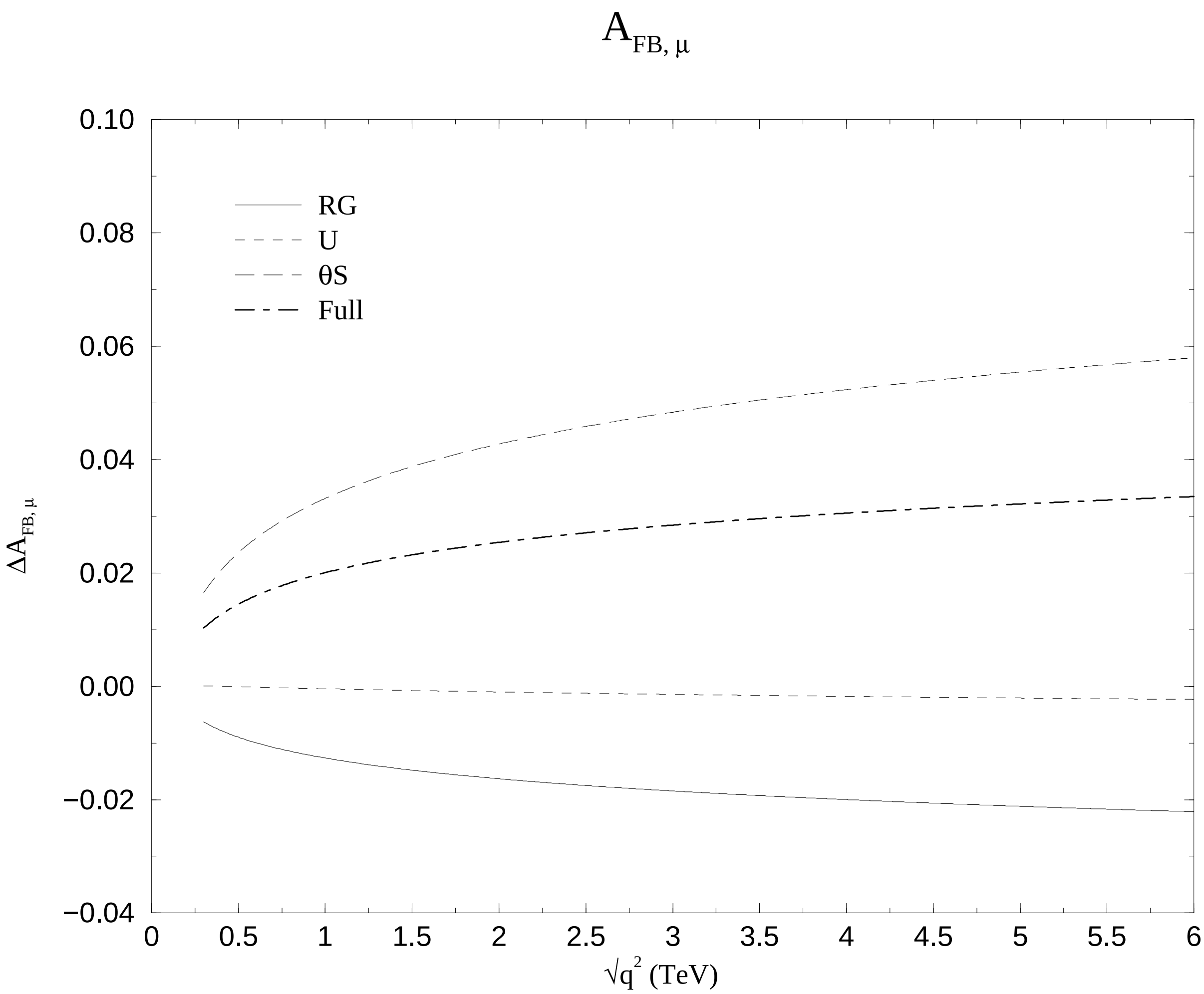

FIG. 3. Separate asymptotic contributions to $A_{FB,\mu}$ as functions of the energy. Same captions as in Fig.2.

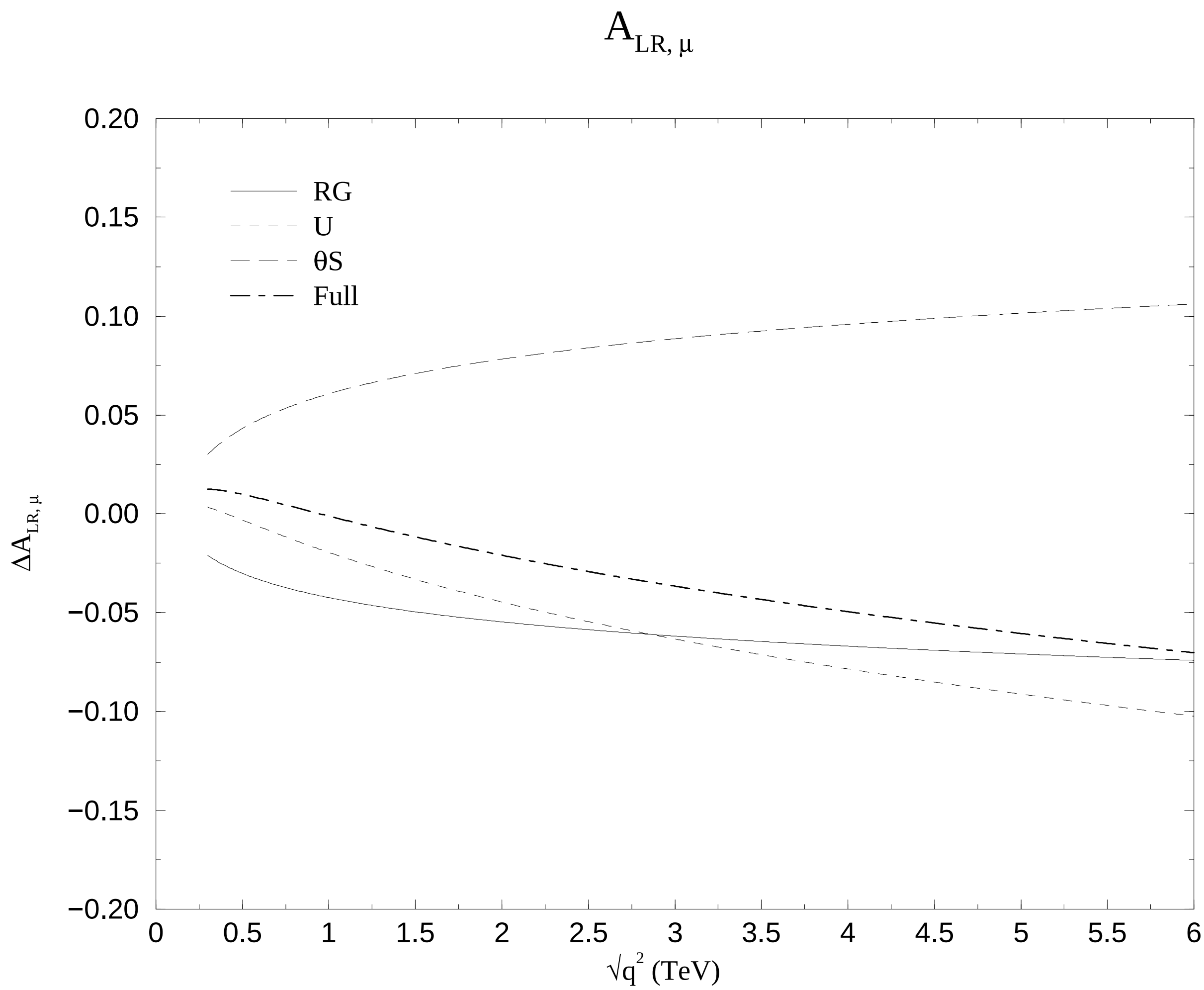

FIG. 4. Separate asymptotic contributions to $A_{LR,\mu}$ as functions of the energy. Same captions as in Fig.2.

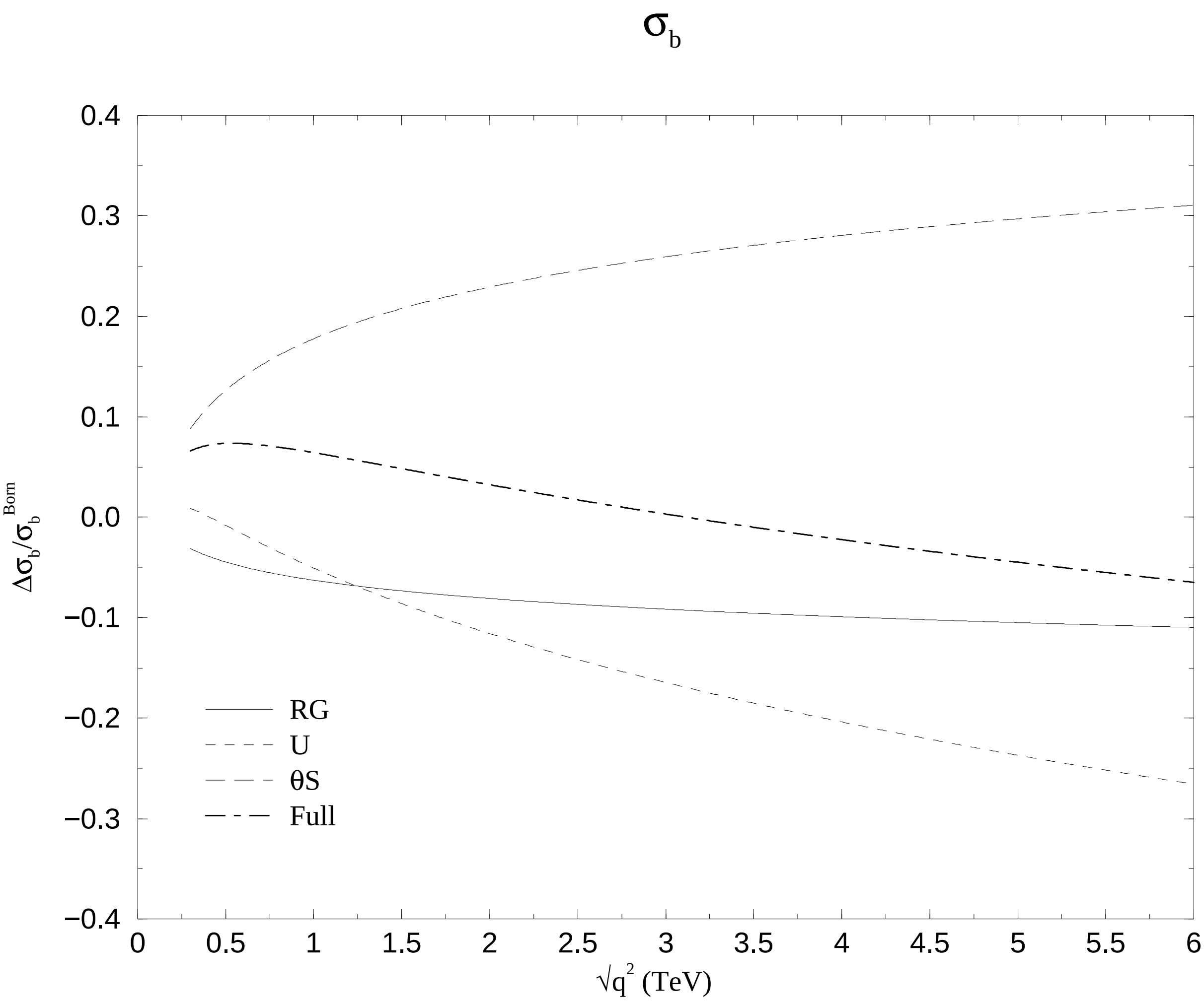

FIG. 5. Separate asymptotic contributions to σ_b as functions of the energy. Same captions as in Fig.2.

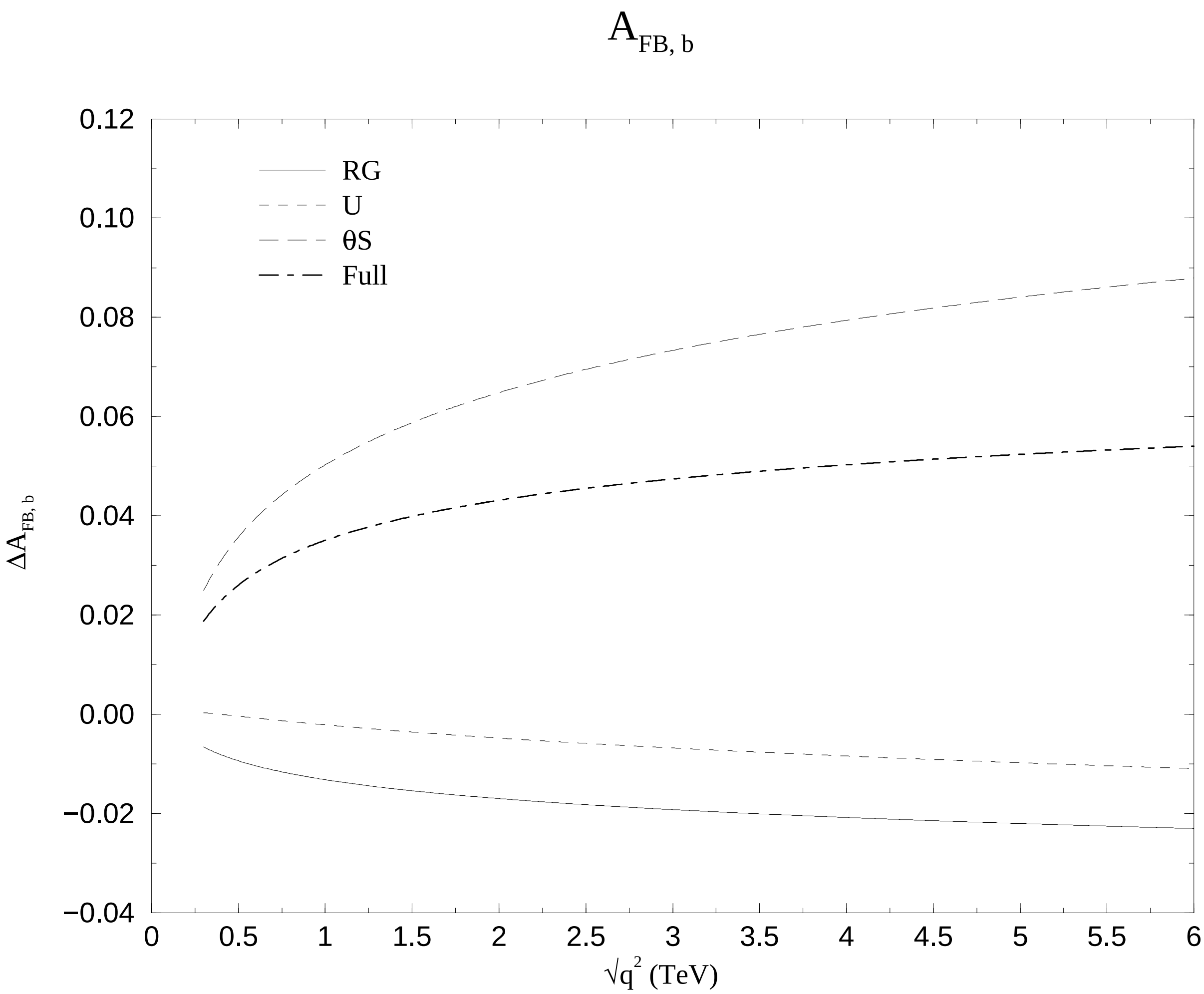

FIG. 6. Separate asymptotic contributions to $A_{FB,b}$ as functions of the energy. Same captions as in Fig.2.

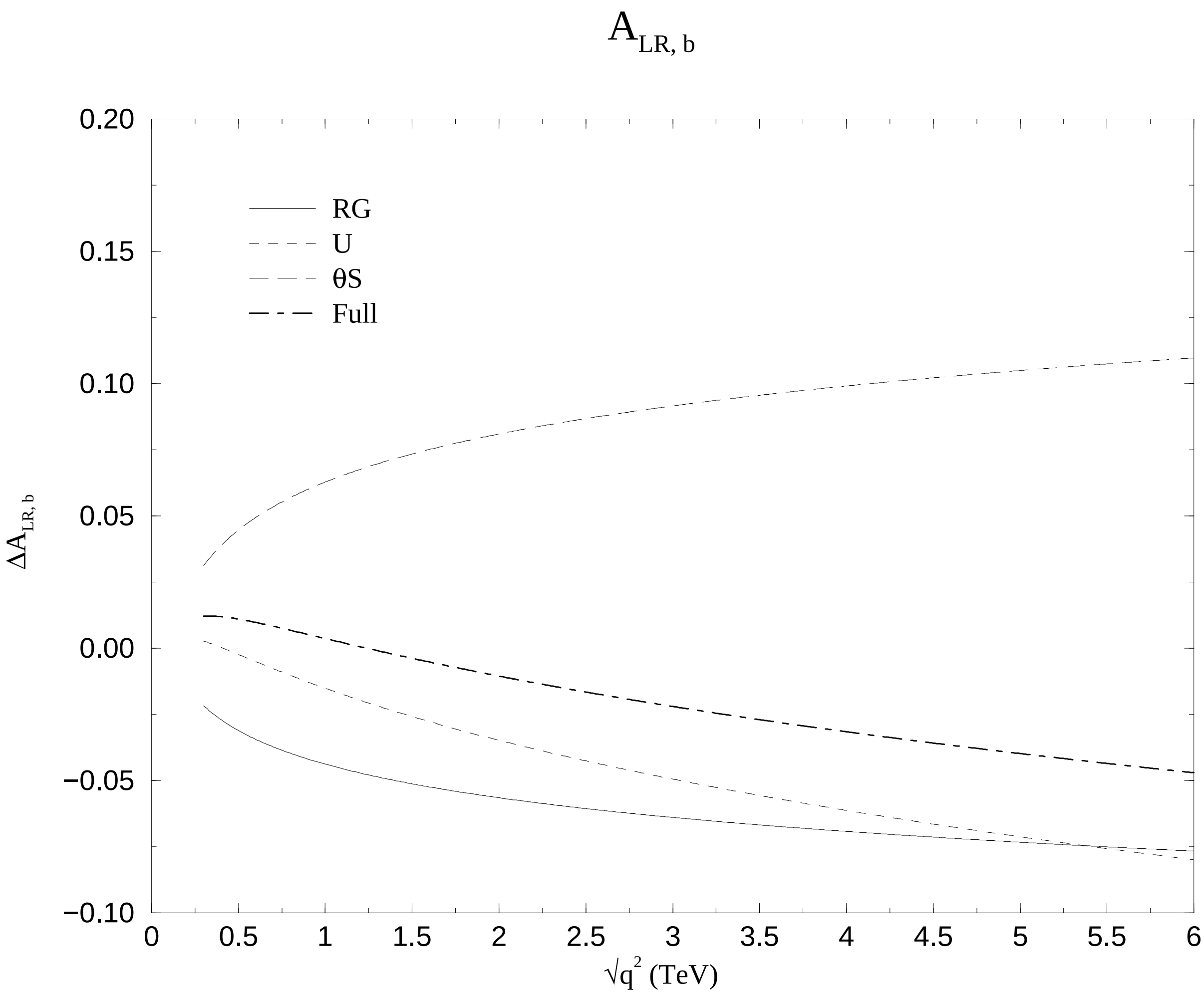

FIG. 7. Separate asymptotic contributions to $A_{LR,b}$ as functions of the energy. Same captions as in Fig.2.

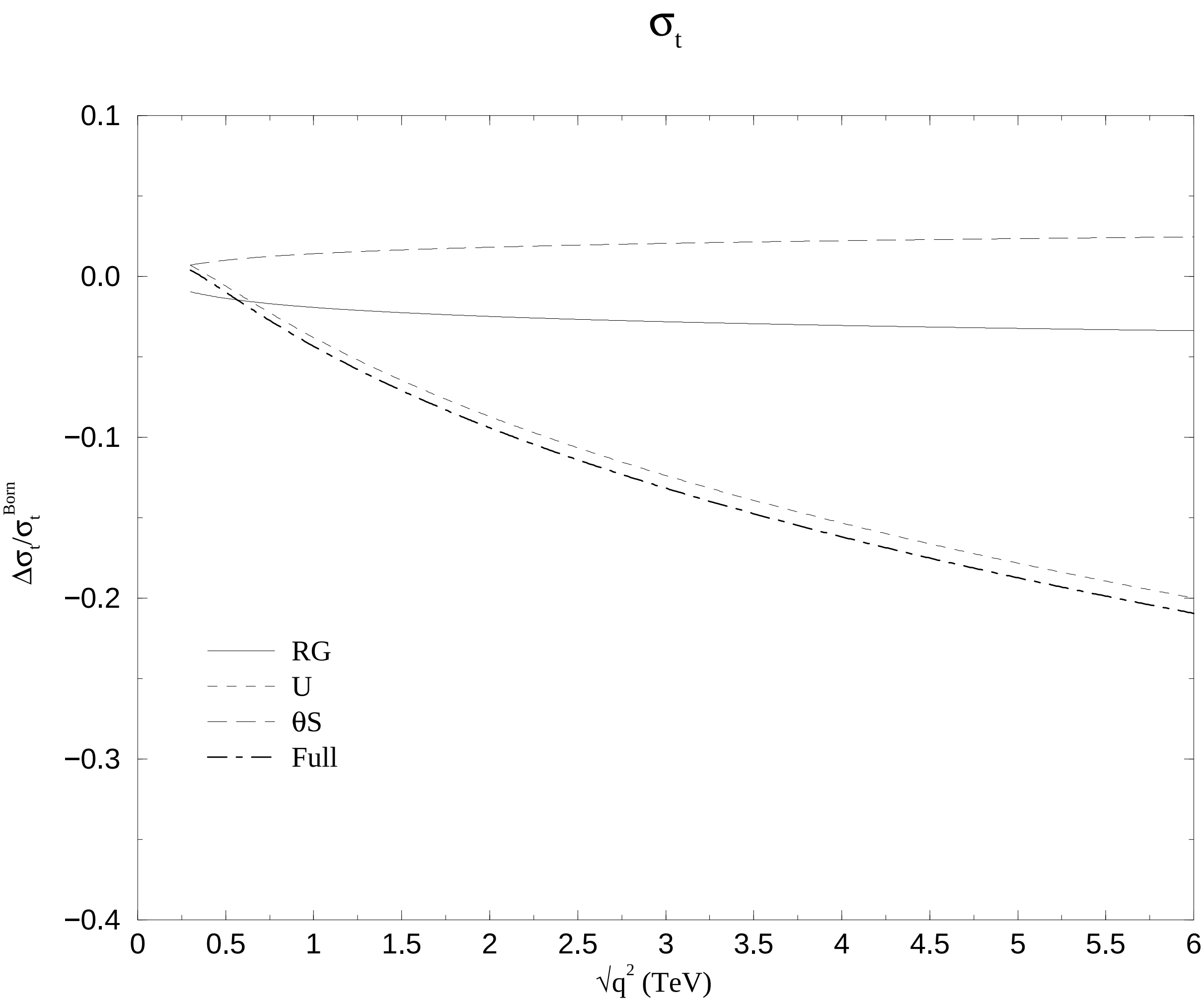

FIG. 8. Separate asymptotic contributions to σ_t as functions of the energy. Same captions as in Fig.2.

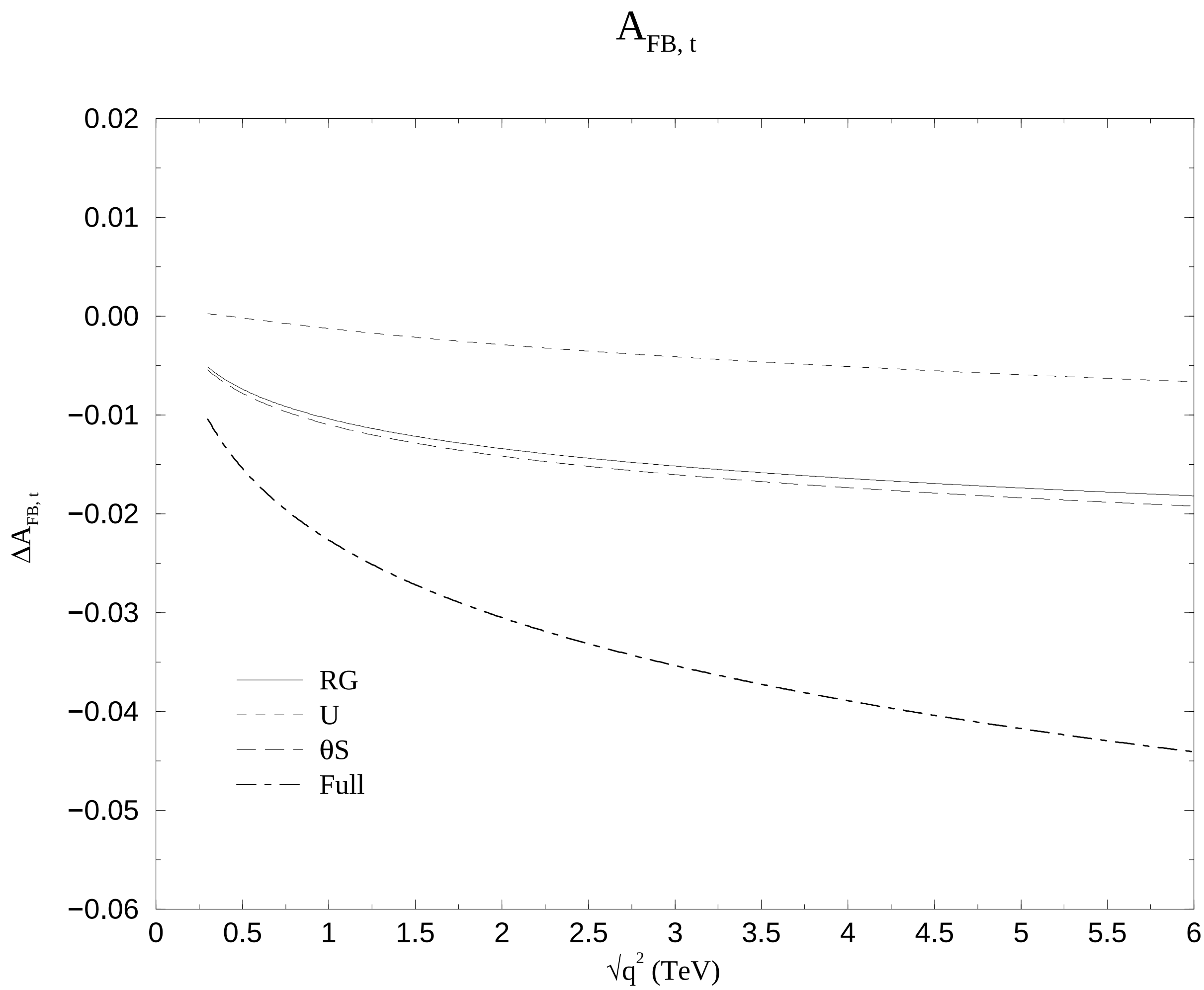

FIG. 9. Separate asymptotic contributions to $A_{FB,t}$ as functions of the energy. Same captions as in Fig.2.

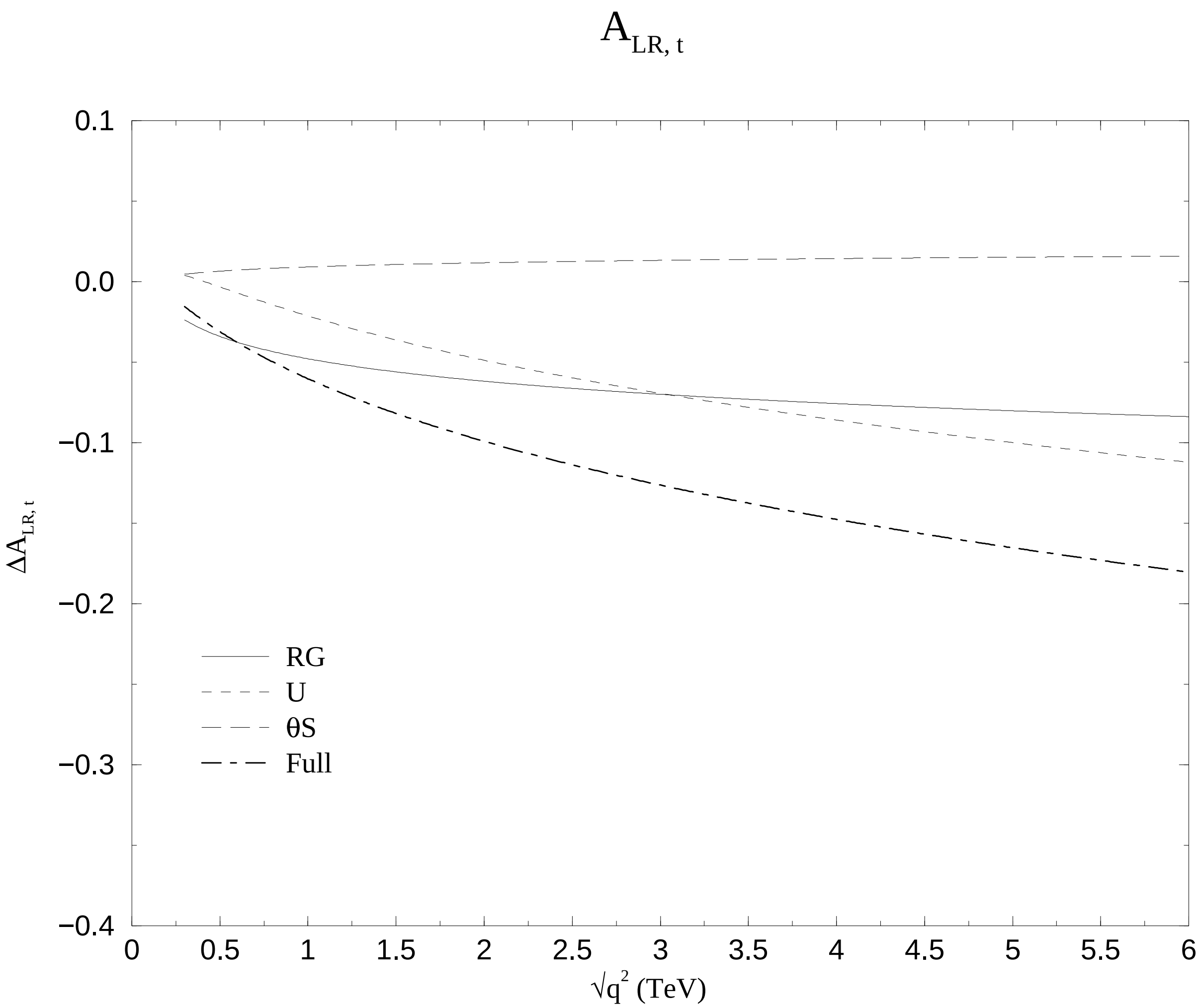

FIG. 10. Separate asymptotic contributions to $A_{LR,t}$ as functions of the energy. Same captions as in Fig.2.

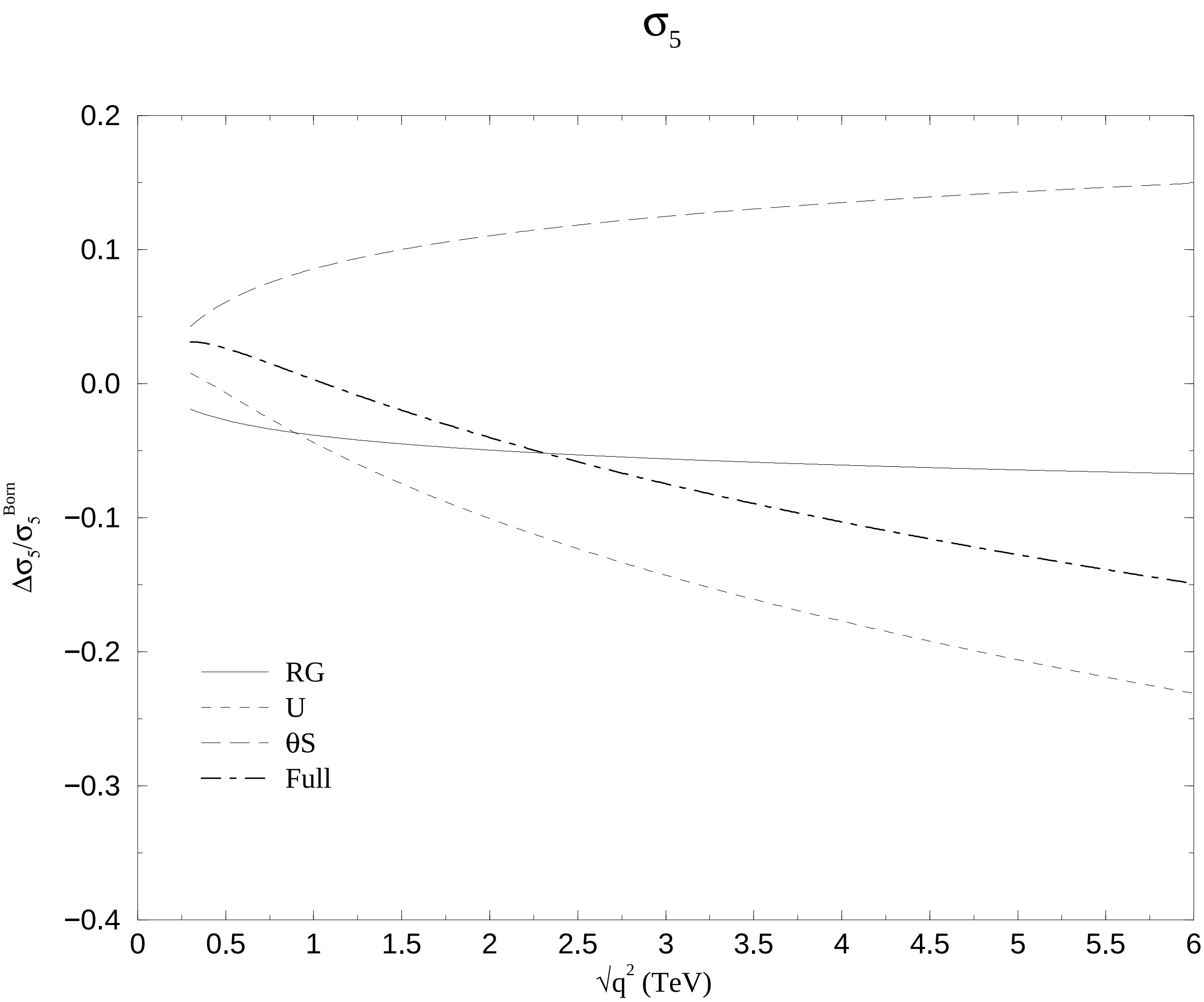

FIG. 11. Separate asymptotic contributions to σ_5 as functions of the energy. Same captions as in Fig.2.

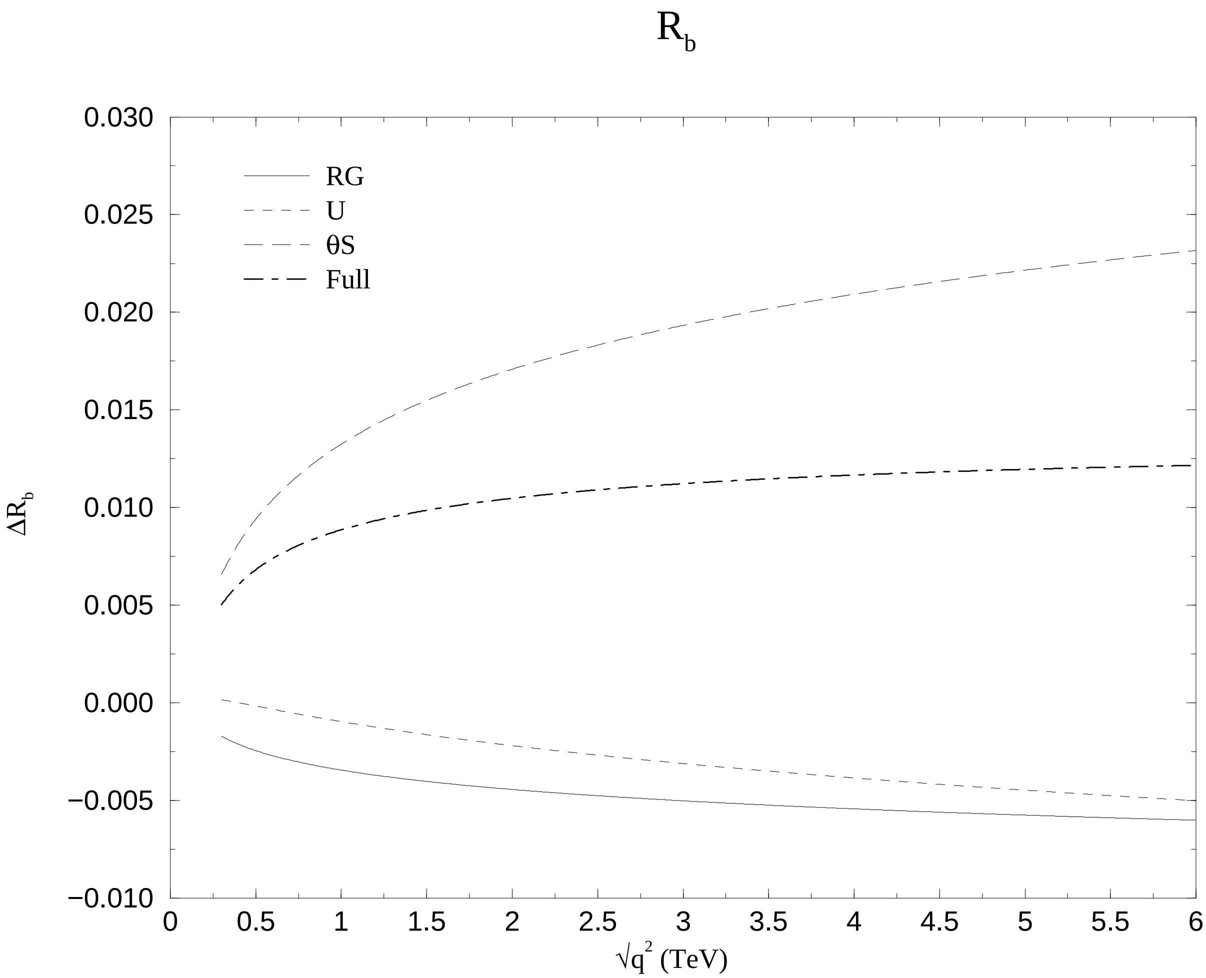

FIG. 12. Separate asymptotic contributions to R_b as functions of the energy. Same captions as in Fig.2.

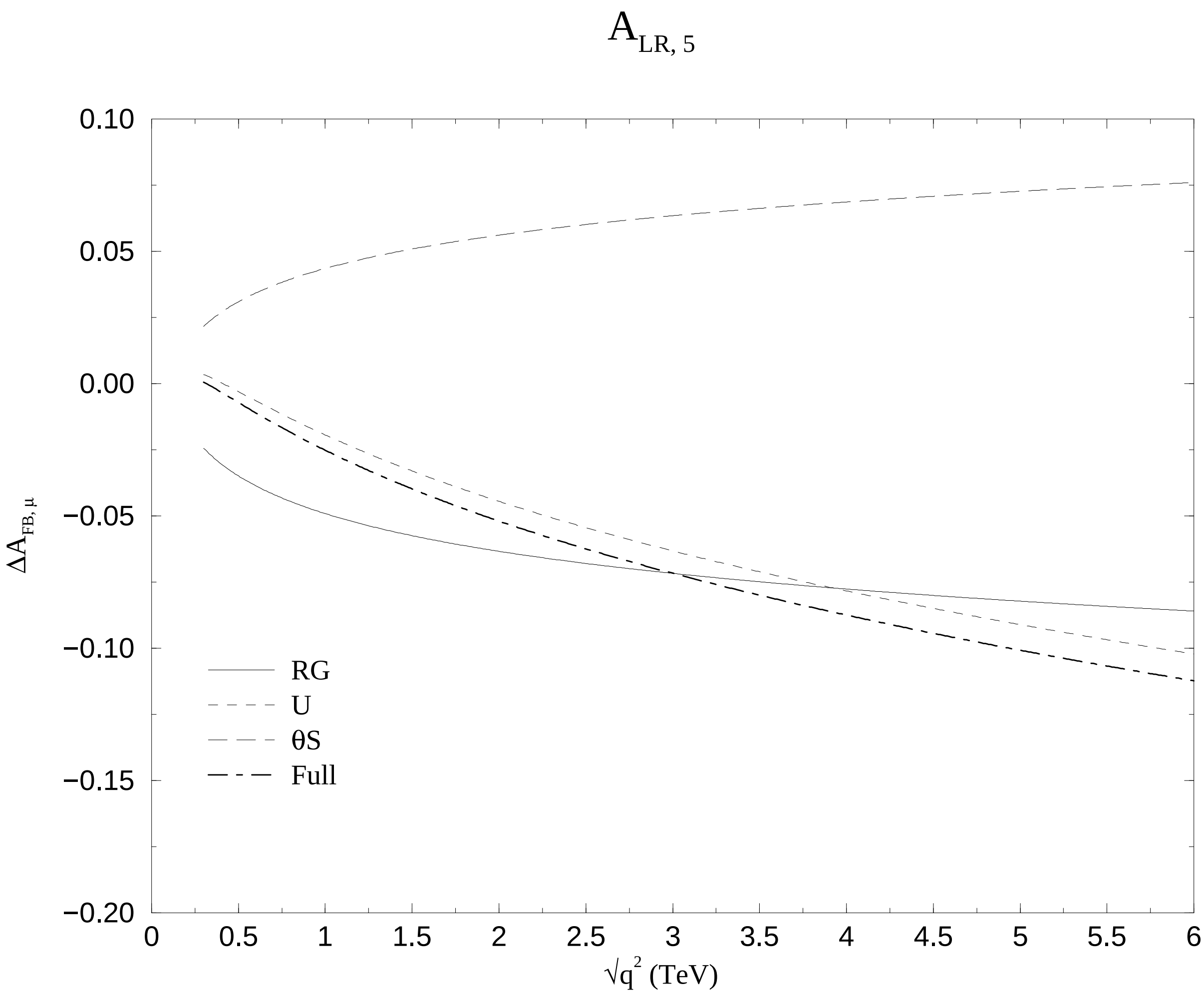

FIG. 13. Separate asymptotic contributions to $A_{LR,5}$ as functions of the energy. Same captions as in Fig.2.